On Global Aspects of Dual invariant theories: M2-brane vs DFT

María Pilar García del Moral (UA)

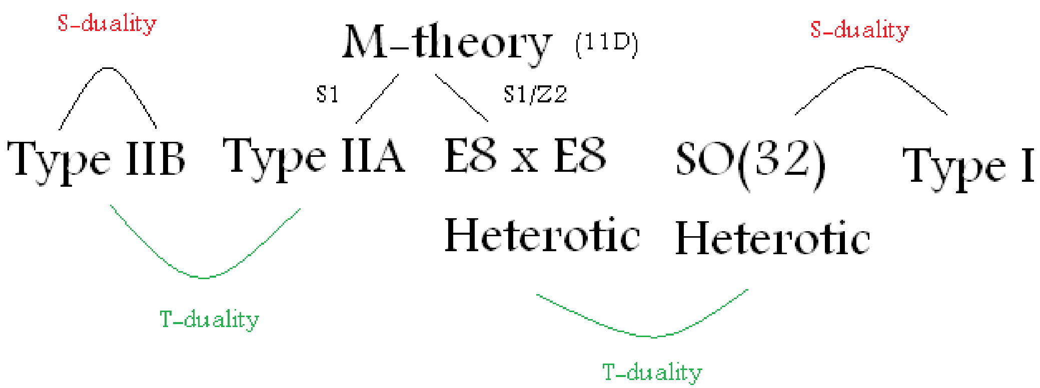
In collaboration with J.M. Peña (U. Central de Venezuela)
A. Restuccia. (U. Antofagasta, Chile)
C. Las Heras (U. Antofagasta,Chile)

Based on ***Arxiv. 1604.02579,*** IJMMP13, JHEP 12, JHEP 1109

2018 Gravity, Holography, Strings and Non commutative Geometry Workshop Belgrade U. ,February 2018

M-theory

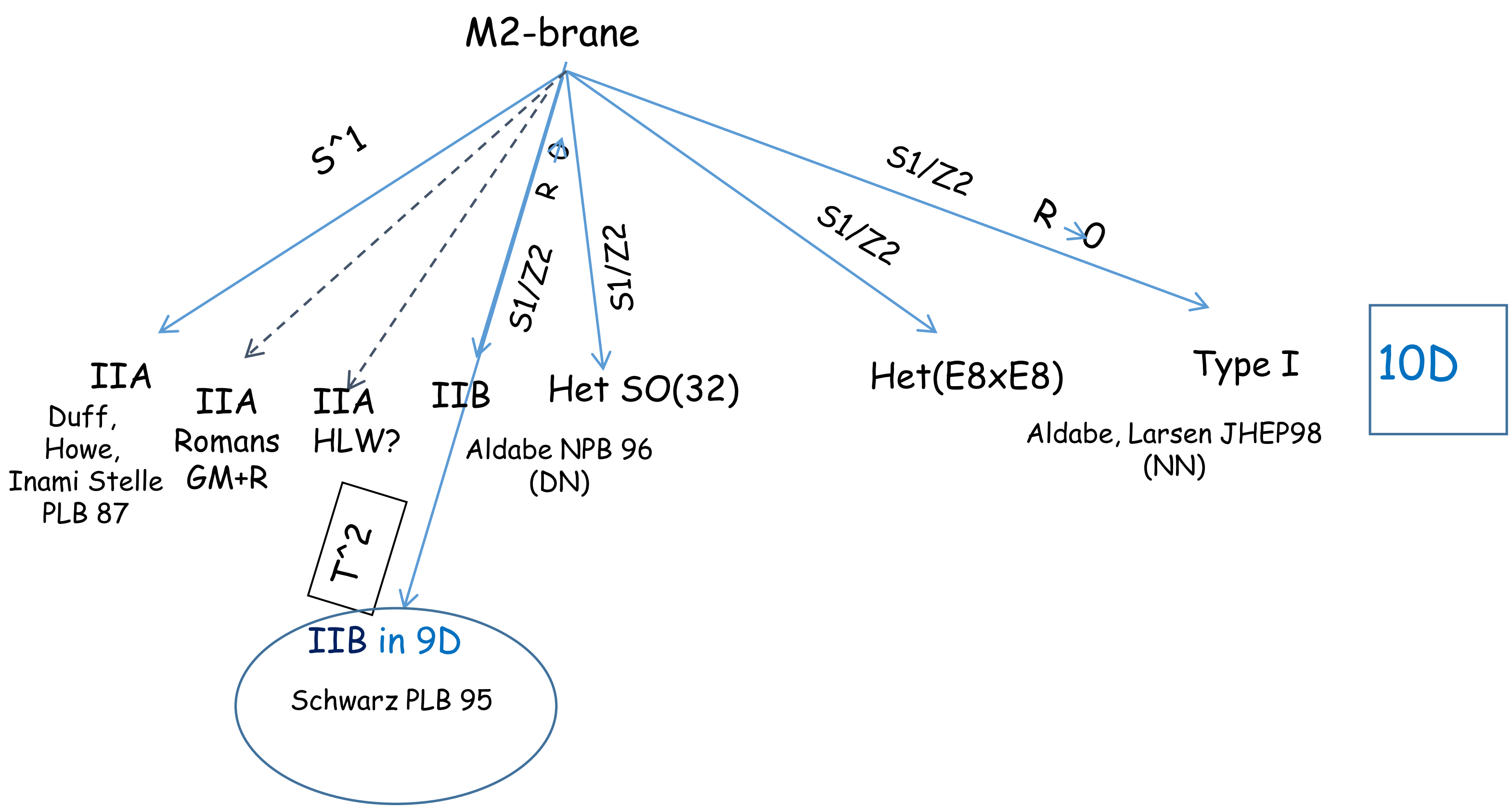
- **M-theory** is conjectured as a 11D quantum theory whose low energy limit is 11D **Supergravity**. It contains **Supermembranes** also called M2-branes, M5- branes, maybe more... **StringTheories** are perturbative limits of the theory in 10D



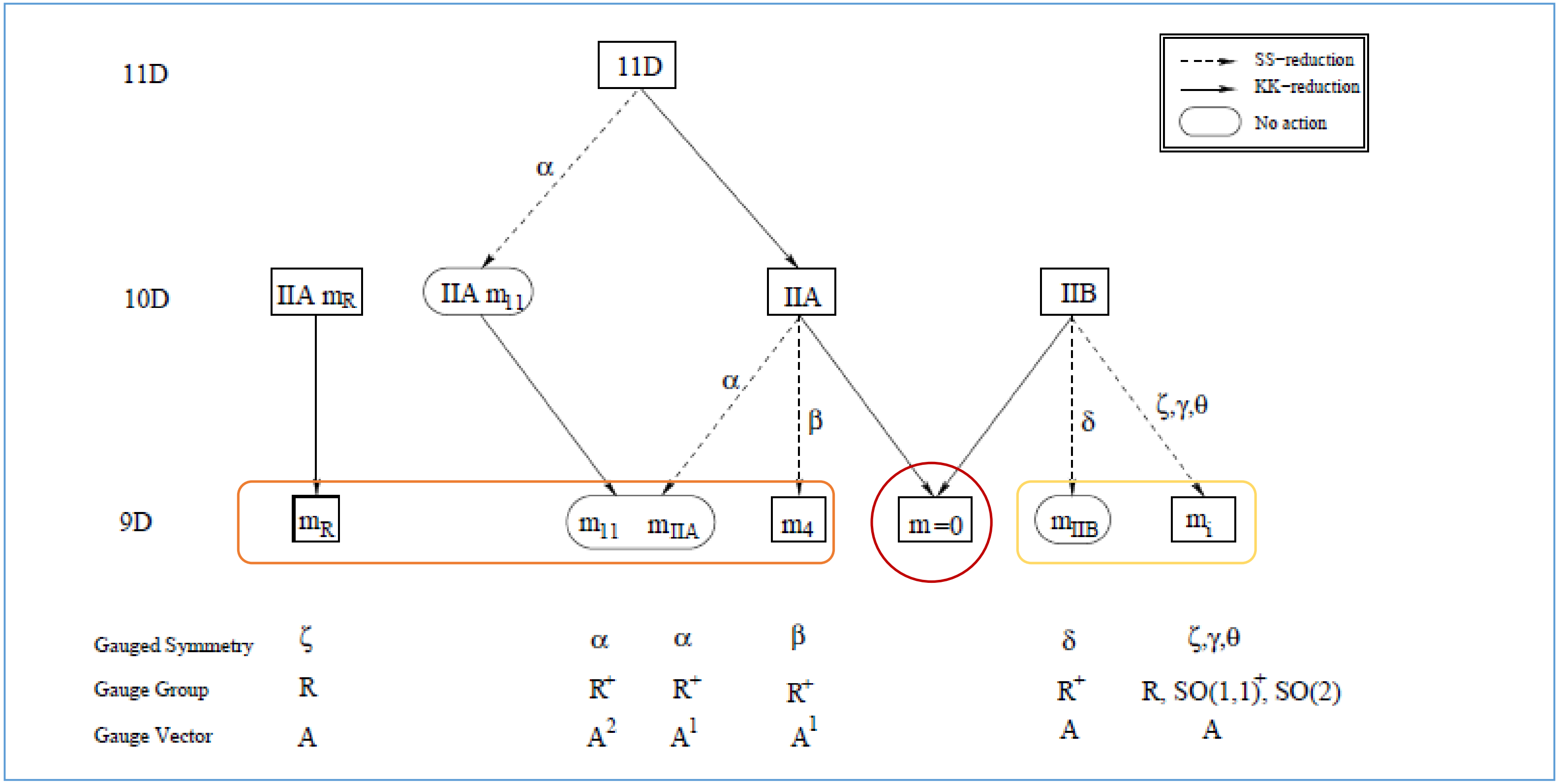
- We do not know the complete quantum description of the theory but we know some corners.
- Let's Look for more insights: As a unification theory it should also contain **Dualities** as symmetries of the theory.

Strings from M2

A M2-brane or Supermembrane is a 2+1 dim object embedded in 11D of the target space.



Type II supergravity in 9D



P. Meessen, T. Ortin 98 ;
Bergshoeff ,T. De Wit, U. Gran, R. Linares, D. Roest 2002
JJ. Fernandez-Melgarejo, T. Ortin, E. Torrente-Lujan, 12

M-theory

Approaches

- **Double Field Theory** (Generalized Geometry) with dualities incorporated in the String/M Effective action
- Supermembrane Worldvolume description since it is an element of M-theory.

In both approaches global description is relevant

String/M-theory invariant actions under dualities? **T-duality** is expected to be realized as a symmetry of those theories when they are defined on **Torus bundles.**

Hull, Dabholkar JHEP 06; Hull JHEP 07

Gauged supergravities are related to SS compactifications **(Monodromies: Arithmetic subgroup of the global symmetry that becomes gauged).**

Hull CQG 04;

Supermembrane worldvolume Theory

- It is a 2+1D object in a 11D target space and its action in a general supergravity background is

Bergshoeff, Sezgin, Townsend PLB 87

$$S = T_3 \int d^3\xi \left[-\frac{1}{2}\sqrt{-g}g^{ij}E_i{}^{\hat{a}}E_j{}^{\hat{b}}\eta_{\hat{a}\hat{b}} + \frac{1}{2}\sqrt{-g} + \frac{1}{6}\varepsilon^{ijk}E_i^{\hat{A}}E_j^{\hat{B}}E_k^{\hat{C}}C_{\hat{C}\hat{B}\hat{A}} \right].$$

- In the L.C.G in 11D flat space its hamiltonian is greatly simplified to

De Wit, Hoppe, Nicolai NPB 87

$$H = T^{-2/3}\int_\Sigma \sqrt{W}\left[\frac{1}{2}\left(\frac{P_M}{\sqrt{W}}\right)^2 + \frac{T^2}{4}\{X^M, X^N\}^2 + \sqrt{W}\bar{\theta}\Gamma_-\Gamma_m\{X^m, \theta\}\right].$$

- Constrained by Super Area Preserving Diffeomorphims.

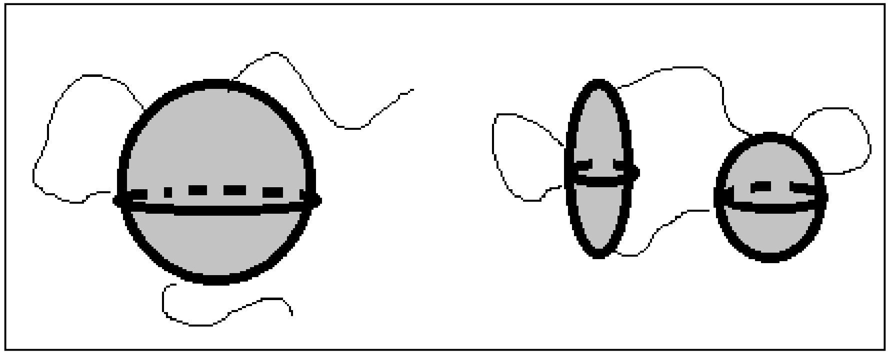
Supermembrane in 11D

- Classically contains String-like spikes at zero cost energy

$$\{X^m(\sigma^1,\sigma^2,\tau),X^n(\sigma^1,\sigma^2,\tau)\}=\frac{\epsilon^{ab}}{\sqrt{W}}\partial_a X^m \partial_b X^n$$

$$V=0 \Leftrightarrow X^m(a\sigma^1+b\sigma^2,\tau)$$

It does not preserve the topology nor the number of particles



- At Quantum level the spectrum is **continuous**

De Wit, Luscher, Nicolai NPB 88

- The groundstate is conjectured to be the wavefunction describing the supermultiplet of 11D supergravity. *Recent advances BGMR

Supermembrane compactified on a Torus

- Impose a winding condition on the maps

$$R \times \Sigma \Rightarrow M_9 \times T^2$$

$$\oint_{C_s} dX = 2\pi R(l_s + m_s\tau)$$
$$\oint_{C_s} dX^m = 0$$

- We impose a topological condition. There are two inequivalent sectors classified atending whether they satisfy it or not:

Martin Torrealba, Restuccia 96

$$\int_\Sigma dX^r \wedge dX^s = 0$$

With n equal to zero

$$\int_\Sigma dX^r \wedge dX^s = n\epsilon^{rs} Area(T^2) \quad r,s = 1,2$$

With n different from zero

Supermembrane with central charges: Non trivial sector

- The hamiltonian is

Martin,Ovalle, Restuccia NPB 00

$$H=\int_\Sigma \sqrt{W}d\sigma^1\wedge d\sigma^2[\frac{1}{2}(\frac{P_m}{\sqrt{W}})^2+\frac{1}{2}(\frac{P^r}{\sqrt{W}})^2+\frac{1}{4}\{X^m,X^n\}^2+\frac{1}{2}(\mathcal{D}_rX^m)^2+\frac{1}{4}(\mathcal{F}_{rs})^2$$
$$+(n^2\mathrm{Area}^2_{T^2})+\int_\Sigma\sqrt{W}\Lambda(\mathcal{D}_r(\frac{P_r}{\sqrt{W}})+\{X^m,\frac{P_m}{\sqrt{W}}\})]$$
$$+\int_\Sigma\sqrt{W}[-\overline{\Psi}\Gamma_-\Gamma_r\mathcal{D}_r\Psi-\overline{\Psi}\Gamma_-\Gamma_m\{X^m,\Psi\}-\Lambda\{\overline{\Psi}\Gamma_-,\Psi\}]$$

Discrete Spectrum

$$\mathcal{D}_rX^m=D_rX^m+\{A_r,X^m\},\ \mathcal{F}_{rs}=D_rA_s-D_sA_r+\{A_r,A_s\},$$

- with

$$D_r=2\pi l_r\theta_r^lR_r\frac{\epsilon^{ab}}{\sqrt{W}}\partial_a\widehat{X}^l\partial_b$$

Boulton, MPGM, Restuccia CQG02, NPB03, NPB08,NPB12

Compact M2 in 9D

(I)

$$\int_\Sigma dX^r \wedge dX^s = 0$$

Continuous Spectrum

Globally Trivial Torus Bundle

(II)

$$\int_\Sigma dX^r \wedge dX^s = n\epsilon^{rs} Area(T^2) \quad r,s = 1,2$$

Discrete Spectrum

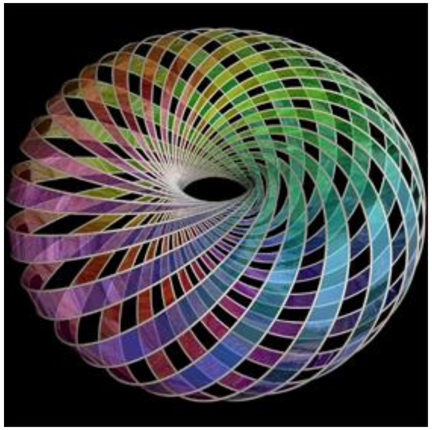
Globally Non trivial Symplectic Torus Bundle with SL(2,Z) monodromy

GM, Martin Peña, Restuccia, JHEP 12

Supermembrane Theory

- Supermembranes couple to 11D Supergravity. It is conjectured that the groundstate could be the 11D Supergravity.
- All the five Superstrings theories can be obtained as a limit of Supermembrane theory.
- Some of the M-theory microscopic degrees of freedom space are described by Supermembranes subject to central charge condition.
- Supermembrane theory compactified on a torus couples to all type of 9D type II supergravities : maximal and all gauged Supergravities.

- BUNDLE DESCRIPTION



Bundle description $F \to E \xrightarrow{\pi} \Sigma$

- The fiber of the bundle **F** corresponds to the target space considered $M_9 \times T^2$ The base of the bundle is the spatial part of the foliated worldvolume of the supermembrane, the Riemann surface of genus 1.

- The fields X are sections of a nontrivial symplectic torus bundle, A is the pull back on the base of the symplectic connection on the fiber bundle.
- The structure group G is the symplectomorphims group preserving the 2-symplectic form of F.

$$M_G : \Pi_1(\Sigma) \to \pi_0(Symp) = SL(2, Z),$$

- We have a torus bundle over a nonflat 2-torus with monodromy.

Bundle description

- **Winding charges** are the elements of the closed one forms associated to the first cohomology class of the base manifold.
- **KK charges** are the components of the different elements of the first homology class of the target space.
- **The transition functions** are connected through the structure group of the symplectomorphisms.
- The action of the supermembrane is a invariant functional defined on the sections of the bundle.
- The supermembrane without imposing the central charge condition and with no monodromy corresponds to a trivial symplectic principal torus bundle over a torus.

Bundle description

- **Monodromies** in SL(2,Z), There are three inequivalent classes: Elliptic, Parabolic and Hyperbolic abelian subgroups of SL(2,Z).
- Supermembrane bundle only couples to the ordinary /gauged supergravity with the same monodromy class.
- According to a Kahn Theorem symplectic torus Bundles are classified by the **coinvariant group for a given monodromy**. There are coinvariants associated to the monodromy class of the fiber and another ones to the base.

$$C_F = \{\mathcal{Q} + (g-1)\hat{\mathcal{Q}}\} \qquad C_B = \{W + (g^* - \mathbb{I})\widehat{W}\}$$

Bundle description

Bundle invariants: Monodromy & Coinvariants

- **Fiber Invariants**, $$(M_G, C_F)$$

with $$M_G : \pi_1(\Sigma) \to \pi_0(G)$$

$$C_F = \{Q + (g - \mathbb{I})\widehat{Q}\}$$

$$M_G = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{\gamma}$$

$$\pi_0(G) = SL(2, Z)$$

$$G = Symp(T^2)$$

$$g \in M_G$$

- **Base Invariants**,

$$(M_G^*, C_B)$$

with $$M_G^* = \Omega M_G \Omega^{-1}$$

$$C_B = \{W + (g^* - \mathbb{I})\widehat{W}\}$$

$$\Omega = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$M_G^* = \begin{pmatrix} a & -b \\ -c & d \end{pmatrix}^{\gamma}$$

$$g^* \in M_G^* = \Omega M_G \Omega^{-1}$$

Dualities

T-duality

- The mass formula becomes

$$M^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2),$$

- It is invariant under the following transformation

$$R \to R' = \frac{\alpha'}{R}, \qquad n \leftrightarrow w.$$

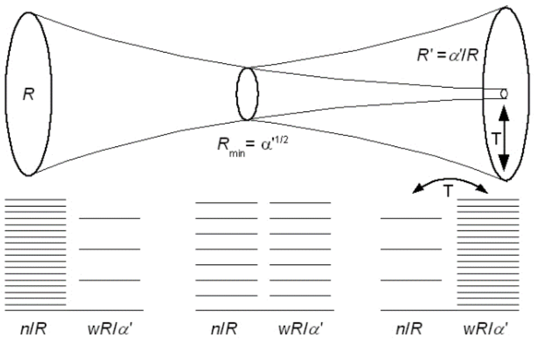


Figure borrowed from P. Candelas Course

T-duality extension in M-theory

The T-duality Transformation,

$$\text{The moduli}: \quad \mathcal{Z}\widetilde{\mathcal{Z}} = 1, \quad \widetilde{\tau} = \frac{\alpha\tau + \beta}{\gamma\tau + \alpha}.$$

$$\text{The charges}: \quad \widetilde{\mathbf{Q}} = \mathcal{T}\mathbf{Q}, \quad \widetilde{\mathbf{W}} = \mathcal{T}^{-1}\mathbf{W},$$

Where we define adimensional variables $\mathcal{Z} = (T_{\mathtt{M2}}\mathtt{A}Y)^{1/3}$

with

$$A = (2\pi R)^2 Im\tau$$

$$Y = \frac{RIm\tau}{|q\tau - p|}.$$

$$\mathcal{T} = \begin{pmatrix} \alpha & \beta \\ \gamma & \alpha \end{pmatrix} \in SL(2, Z)$$

$$\mathbf{Q} = \begin{pmatrix} p \\ q \end{pmatrix} \qquad \mathbf{W} = \begin{pmatrix} l \\ m \end{pmatrix} \in H^1(\Sigma)$$

T-duality extension in Supermembrane-theory

The radius in the dual variables transforms as

$$\widetilde{R} = \frac{|\gamma\tau + \alpha||q\tau - p|^{2/3}}{T^{2/3}(Im\tau)^{4/3}(2\pi)^{4/3}R},$$

The mass operator identity

$$M^2 = T^2 n^2 A^2 + \frac{m^2}{Y^2} + T^{2/3}H = \frac{1}{\widetilde{Z}^2}(\frac{n^2}{\widetilde{Y}^2} + T^2 m^2 \widetilde{A}^2) + \frac{T^{2/3}}{\widetilde{Z}^8}\widetilde{H}.$$

The transformation becomes a **symmetry** by imposing

$$\widetilde{Z} = Z = 1 \Rightarrow T_0 = \frac{|q\tau - p|}{R^3(Im\tau)^2}.$$

a relation between the tension of the supermembrane , the moduli of the target compactification and the KK charges.

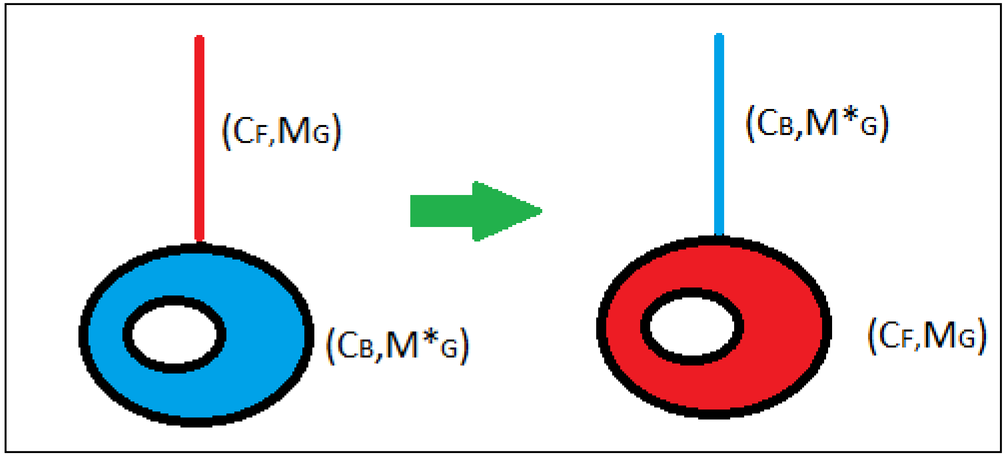
T-duality on Symplectic Bundles

- T-duality transforms moduli, charges, *and* also geometric structures

- The structure of the bundle is transformed interchanging the cohomological classes of the base (Winding) into the homological classes of the torus bundle (Kaluza Klein) and viceversa.

- It maps bundles into bundles with monodromies in the same equivalence class but not necessarily in the same coinvariant equivalence class.

T-duality on Symplectic Bundles



Equivalence classes of bundles transform under T-duality

$$(C_F, C_B) = (\widetilde{C}_B, \widetilde{C}_F)$$

- The monodromy group transforms in the dual bundle to

$$M_G \longrightarrow M_G^* = \Omega M_G \Omega^{-1}$$

T-duality on Symplectic Bundles

- Trivial Monodromy : coinvariants (C_F, C_B) have only one element, Q and W respectively,

$$M_G = \mathbb{I} \qquad\qquad W = \mathcal{T}Q$$

- Non trivial Monodromy : T-duality is realized linearly and non-linearly. Non linear ones imply a change in the gauging group at low energies.

$$Q \xrightarrow{\mathcal{T}} W = \mathcal{T}Q\,, \qquad M_G \xrightarrow{\Omega} \Omega M_G \Omega^{-1}$$

$$(C_F, C_B) = (\widetilde{C}_B, \widetilde{C}_F)$$

M2-brane bundle classification

Clasification of M2-brane bundles

- There are three inequivalent subgroups of the Monodromy group which are a linear representation of the group: Elliptic, Hyperbolic, and Parabolic.

- M2-brane Parabolic bundles are mapped linearly under global T-duality action and the coinvariant classes are preserved.

$$M_p = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \qquad C_p^B = \{\mathbb{T}(Q + (M_p - 1)\hat{Q})\} = \mathbb{T}C_p^F = \widetilde{C}_p^F$$

- At low energies the dual of a parabolic gauged supergravity in 9D corresponds to the parabolic gauged supergravity in the dual sector.

Clasification of M2-brane bundles

- Elliptic and Hyperbolic M2-brane torus bundles

$$M_{Z_3} = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}^{\gamma}, \quad M_{Z_4} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^{\gamma}, \quad M_{Z_6} = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}^{\gamma} \quad M_h = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{\gamma}$$

- At low energies they are in correspondence with the Elliptic and Hyperbolic gauged supergravities.

- T-duality acts nonlinearly in this case

$$\boxed{\mathbb{T}(C_f, C_B) = (\mathbb{T}C_F, \mathbb{T}C_B) \neq (\widetilde{C}_B, \widetilde{C}_F)}$$

Clasification of M2-brane bundles

$$\begin{cases} [\mathbb{T}, M_p] = 0 & \widetilde{C}_B = \mathbb{T} C_F \\ [\mathbb{T}, M_{e,h}] = R & \widetilde{C}_B = \mathbb{T} C_F + R\mathbb{T}^{-1} \\ \text{with} & R = ct\mathbb{B} + ut\mathbb{A} \end{cases}$$

Linear action

Non linear action

Clasification of M2-brane bundles

Elliptic and Hyperbolic M2-brane torus bundles

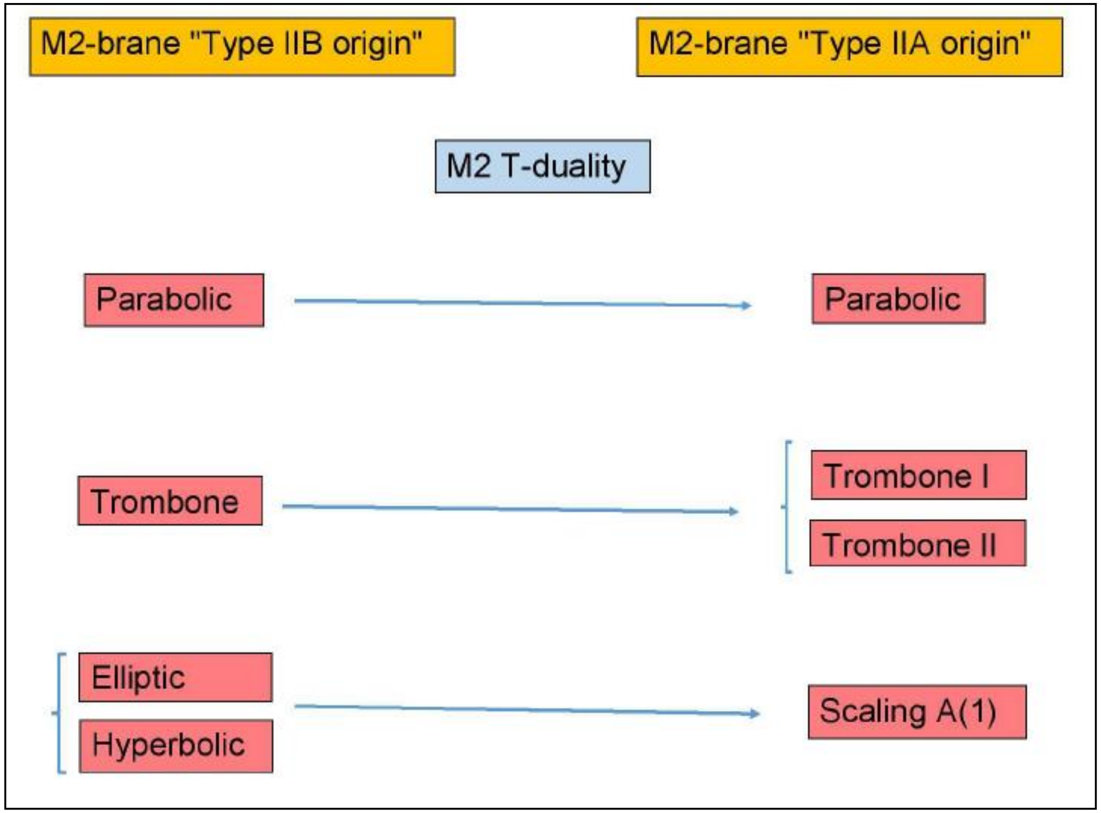
- The Monodromy generators with the parabolic T-duality generator form an algebra. It contains as a subgroup the A(1) algebra generators of collinear transformations (scaling and shifts in one dimension).

$$[A,B]=2A \qquad A=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix} \text{ and } B=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$$

- The T duals of the type II Elliptic and hyperbolic M2-brane bundles are mapped into an inequivalent coinvariant class. The dual monodromy contains two terms, linear and a non-linear one constructed in terms of A(1) generators. We conjecture that at low energies there is a spontaneous symmetry breaking and this effective monodromy is the gauging realized.

Classification of M2-brane bundles duals

- These are the precise T-duality relation between the inequivalent classes of M2-brane bundles



Double Field Theory

Siegel, Tseytlin, Duff, Hull,Zwiebach

The bosonic action of the string is given by

$$S = -\frac{1}{4\pi}\int_0^{2\pi} d\sigma \int d\tau \Big(\sqrt{\gamma}\gamma^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j G_{ij} + \epsilon^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j B_{ij}\Big)$$

Whose associated hamiltonian is

$$H = \frac{1}{2} Z^T \mathcal{H}_G(E) Z + N + \bar{N}$$

With a generalized metric and charges defined as,

$$\mathcal{H}_G(E) = \begin{bmatrix} G_{ij} - B_{ik}G^{kl}B_{lj} & B_{ik}G^{kj} \\ -G^{ik}B_{kj} & G^{ij} \end{bmatrix} \qquad Z = \begin{pmatrix} \omega^i \\ p_i \end{pmatrix}$$

Defined in terms of a background matrix

$$E_{ij} \equiv G_{ij} + B_{ij} = \begin{bmatrix} E_{mn} & 0 \\ 0 & g_{\mu\nu} \end{bmatrix} \qquad E_{mn} = G_{mn} + B_{mn}$$

O(D,D) invariance

The action is invariant under O(D,D) transformations,
the T-dual symmetry group,

$$\mathcal{H}_G(E') = h\,\mathcal{H}_G(E)\,h^T \quad \text{with} \quad h\eta h^T = \eta$$

And $h \in O(n,n,\mathbb{Z})$

$$\eta = \begin{pmatrix} 0 & \mathbb{I}_{D\times D} \\ \mathbb{I}_{D\times D} & 0 \end{pmatrix}$$

To mimic this behaviour extending Supergravity to Double Field Theory, one extends the coordinates to include the canonical conjugated coordinates of the winding modes.

$$X^M = (\tilde{x}^i, x_i)$$

Analogously one extends the derivatives

$$\tilde{\partial_M} = \left(\frac{\partial}{\partial \tilde{x}_i}, \frac{\partial}{\partial x^i}\right)$$

and other mathematical structures like the Lie bracket to a Courant Bracket…

O(D,D) Invariant actions

The generalized Einstein Hilbert action is

$$S = \int d^{2D}X e^{-2d}\mathcal{R},$$

With a generalized curvature

$$e^{-2d} = \sqrt{g}e^{-2\phi}$$

$$\begin{aligned}\mathcal{R} &= 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\delta_N d,\\ &+ \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL}.\end{aligned}$$

Subject to a strong condition to match the correct number of degrees of freedom

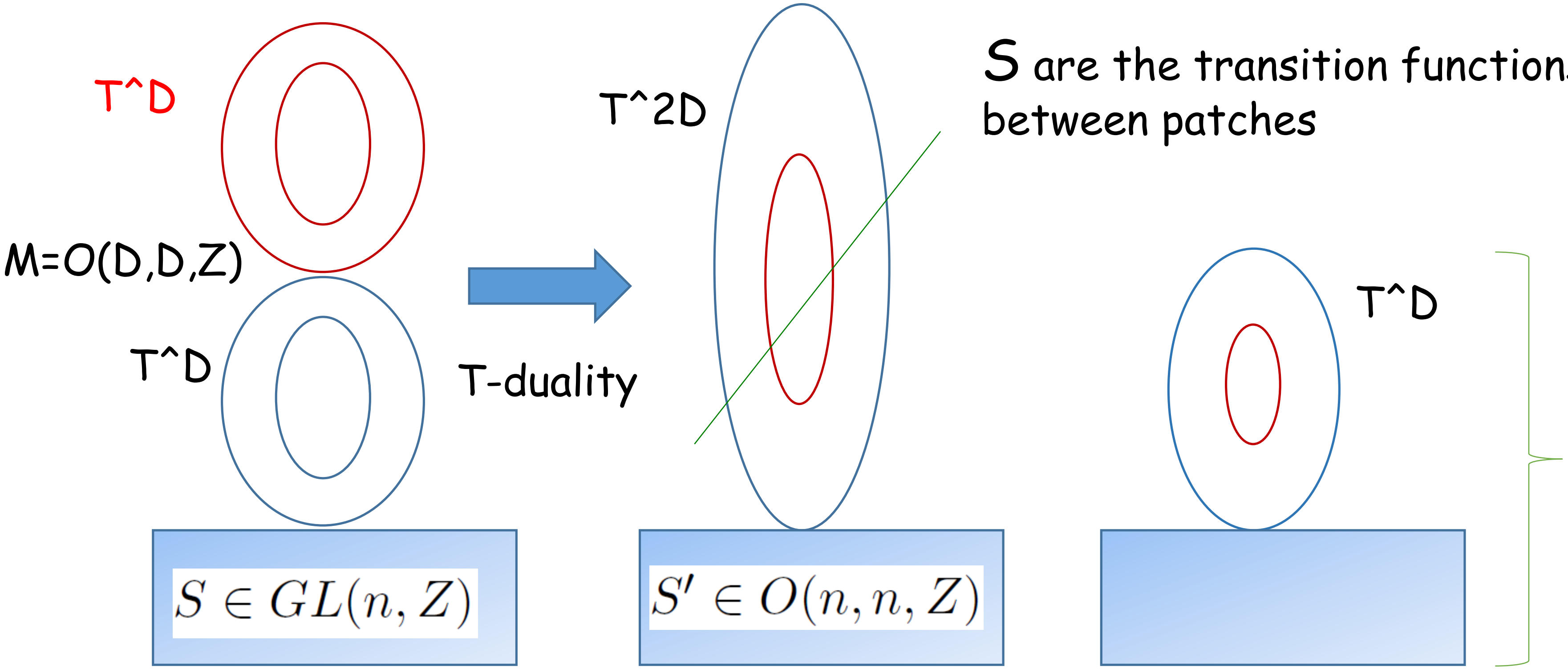
$$\eta^{MN}\partial_M(A)\partial_N(B) = \partial^M(A)\partial_M(B) = \partial^M\partial_M(AB) = 0$$

With the fields satisfying a generalized lie algebra condition

$$\xi^M = (\tilde{\lambda}_i, \lambda^i)$$

$$\begin{aligned}\mathcal{L}_\xi A_M &\equiv \xi^P\partial_P A_M + (\partial_M\xi^P - \partial^P\xi_M)A_P\\ \mathcal{L}_\xi B^M &\equiv \xi^P\partial_P A^M + (\partial^M\xi_P - \partial_P\xi^M)B^P\end{aligned}$$

Global T-duality aspects of DFT



Geometric vacua

T-folds

Non geometric vacua

Lust et al 15

Example $T^4 = T^2 \times T^2$

τ

$\rho = b + iA$

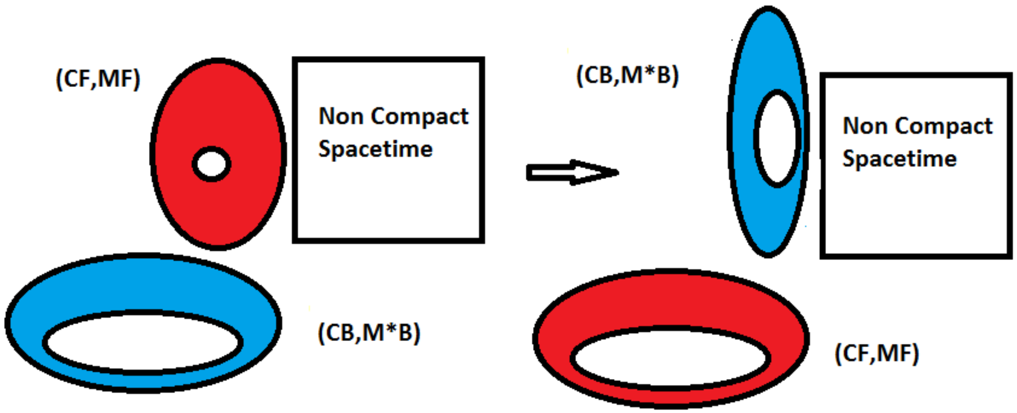
T-duality

$O(2, 2, \mathbb{Z})$

$SL(2, Z)_\tau \times SL(2, Z)_\rho \times Z_2 \times Z_2$

$\tau \Leftrightarrow \rho, \quad (\tau, \rho) \to (-\overline{\tau}, -\overline{\rho})$

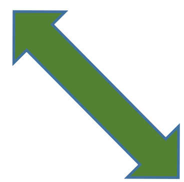
Comparison

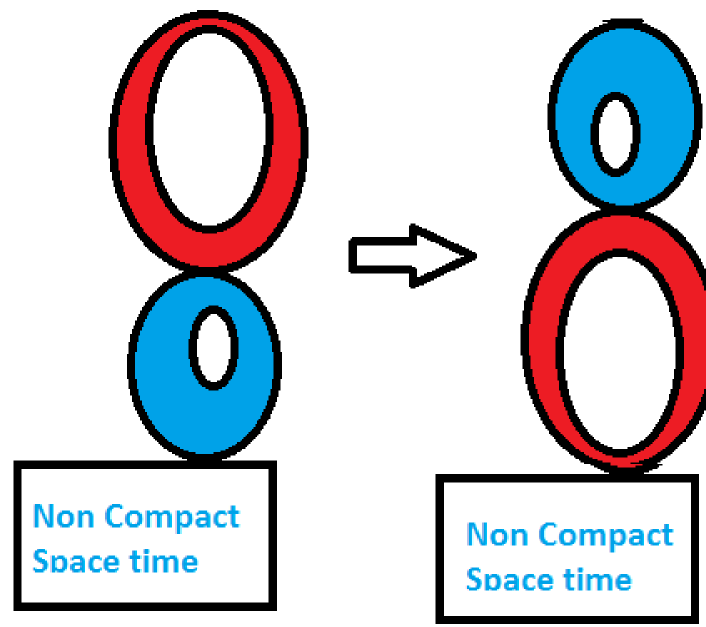


Geometry is not interchanged
Vacua is always geometrical
Monodromy is in GL(2,Z)
Global sym

$$SL(2,Z)_\Sigma \times SL(2,Z)_{T^2} \times Z_2$$

Monodromy=O(D,D,Z)





Polarization interchanges geometries
Monodromy is on O(D,D,Z)
Generically vacua is not geometrical
Theory invariant under O(D,D)
In particular

$$SL(2,Z)_\tau \times SL(2,Z)_\rho \times Z_2 \times Z_2$$

Conclusions

- Supermembrane symplectic torus bundles with monodromy in SL(2,Z) are classified according to their coinvariants.
- T-duality acts as a symmetry at the level of the mass operator in all cases.
- At global level, T-duality interchanges the cohomological charges of the base manifold into the homological charges of the fiber with the dual monodromy equivalence class.

- The gauging of the type II supergravities in 9D can be completely explained in terms of the M2-brane bundles coinvariant classification.

- Only parabolic supermembrane torus bundle are T-dual invariant locally and globally.

- There are certain signals of connection in the global aspects of between DFT in a factorized torus with monodromy in O(2,2,Z) and the supermembrane on a non trivial symplectic torus bundle with monodromy in such a way that could may think that a deeper connection between both theories. It is work in progress.

THANKS!

T-duality String limit

GM, Peña, Restuccia, IJGMMP 13

- Supermembrane

T-duality on the supermembrane M9xT2

$$\text{The moduli}: \quad \mathcal{Z}\widetilde{\mathcal{Z}} = 1, \quad \widetilde{\tau} = \frac{\alpha\tau + \beta}{\gamma\tau + \alpha};$$

$$\text{The charges}: \begin{pmatrix} \widetilde{p} \\ \widetilde{q} \end{pmatrix} = \Lambda_0 \begin{pmatrix} p \\ q \end{pmatrix}, \begin{pmatrix} \widetilde{l}_1 & \widetilde{l}_2 \\ \widetilde{m}_1 & \widetilde{m}_2 \end{pmatrix} = \Lambda_0^{-1} \begin{pmatrix} l_1' & l_2' \\ m_1' & m_2' \end{pmatrix}$$

String-like Config.

String-like Config.

IIB String mass operator

IIA String mass operator M9xS1

T-duality of Closed String

$$R \leftrightarrow \alpha'/R \qquad n \leftrightarrow w$$

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}\left(N + \tilde{N} - 2\right)$$

Supergravity reductions

- **Kaluza Klein reduction** $\hat{\phi}(x,z) = \sum_n \phi_n(x)\, e^{i\,n\,z/L}$

$\hat{\Box}\,\hat{\phi} = 0 \;\Rightarrow\; \Box\phi_n - \frac{n^2}{L^2}\,\phi_n = 0$ Truncate to zero mode

- **Scherk-Schwarz Compactifications (Twisting)**:
- Suppose there is a Global symmetry G acting on the scalar fields

$\phi \longrightarrow g(\phi)$ $\boxed{\phi(x^\mu, y) = g_y(\phi(x^\mu))}$ $g(y) = \exp(My)$ Scherk, Schwarz PLB79

- The map g is not periodic but has a **Monodromy.** It has sense in a **Bundle description**

$\boxed{\mathcal{M}(g) = \exp M}$ with $M = g^{-1}\partial_y g$

Type II gauged Supergravities in 9D

- N=2 type II Supergravity matter content:(three scalars, three gauge fields, 2 antisymmetric 2-forms,the three form the vielbein, 2 dilatinos and one spinor) has Global Symmetries GL(2,R).
- When some of the global symmetries **G become gauged**, take for example IIB sector, the transformation laws for the gauge fields become modified by the Monodromy group contained in GL(2,R).

$$\vec{A} \to \vec{A} - d\lambda, \qquad \vec{B} \to \mathcal{M}(\vec{B} - \vec{A}d\lambda)$$

- It is induced a covariant derivative associated to the particular gauging considered.

$$\partial_\mu \bullet \to D_\mu \bullet = \partial_\mu \bullet + g\Theta[A_\mu, \bullet]$$

- **There are eight inequivalent typeII gauged supergravities in 9D, classified as Elliptic, Hyperbolic, Parabolic, A(1) and Trombone.**