Brussels sprouts, black hole mass and pre-holography

Rodrigo Olea

Universidad Andrés Bello, Chile

with G.Giribet (NYU), O. Miskovic (PUCV) and D.Rivera-Betancour (UNAB)

Workshop on Gravity, Holography, Strings and Noncommutative Geometry
February 1st 2018, Belgrade

A mathematical game: sprouts

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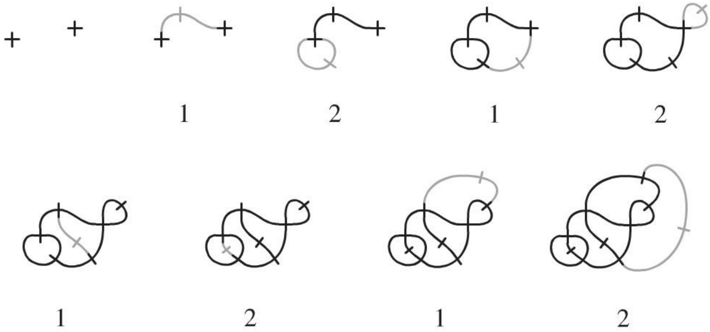
- *For n seeds, the game will last at most* $(3n-1)$ *moves.*

A game of no chance: Brussels sprouts



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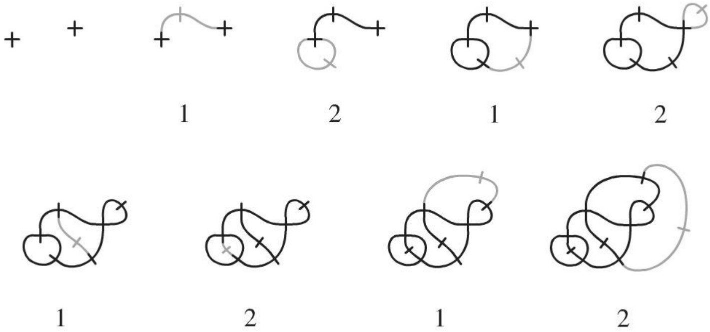

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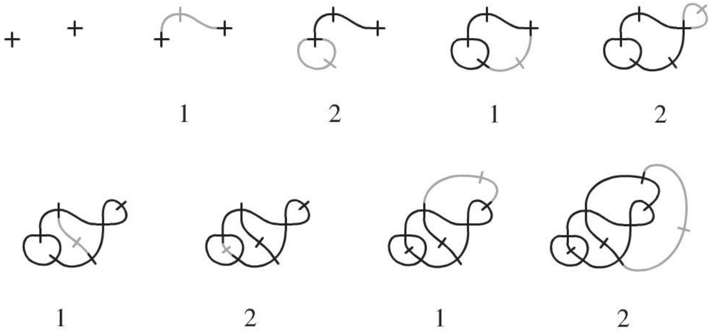

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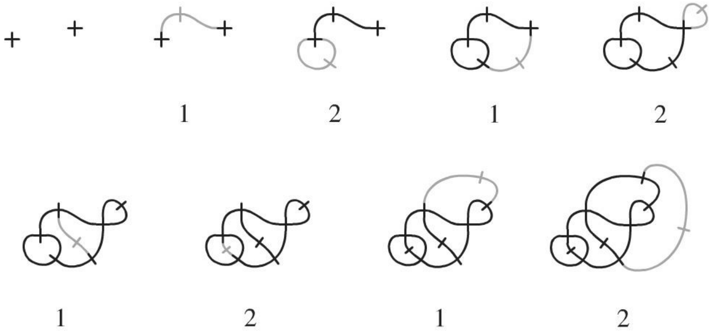

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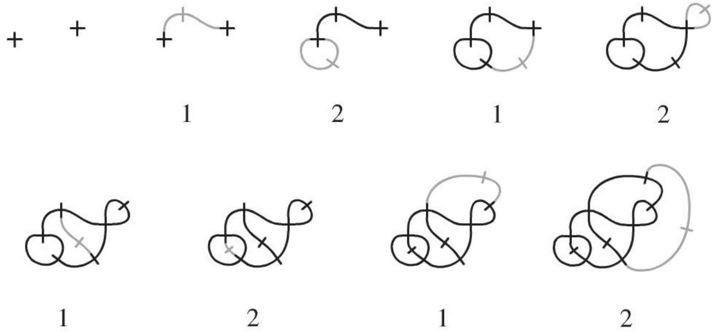

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- *if n is odd, the first player wins*
- *if n is even, the second player wins*

- **Final graph for Brussels sprouts,** i = 10, e = 16, r = 8.

Planar Graphs and the Euler Characteristic

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$$\chi = i - e + r$$

- $\chi = 2$ **for any planar graph**

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Topological Invariants: 4D Electrodynamics

- **Pontryagin term is quadratic in F**

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- **It is a topological invariant (locally a boundary term) such that**

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- **Total Action**

$$\tilde{I} = -\frac{1}{4}\int\limits_M dtd^3x\left(F^{\mu\nu}F_{\mu\nu} + \gamma\ {}^*F^{\mu\nu}F_{\mu\nu}\right).$$

- **Bianchi identity $\Rightarrow$ the last term does not contribute to the *EOM*.**

Topological Invariants: 4D Electrodynamics

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- **Additional surface terms may modify the boundary conditions**

$$\delta I = \int_M d^4x\, \partial_\mu F^{\mu\nu} \delta A_\nu - \int_{\partial M} n_\mu \left(F^{\mu\nu} + \gamma \ \ast F^{\mu\nu} \right) \delta A_\nu$$

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- **Alternative: asymptotic (anti) self-duality in $F^{\mu\nu}$**

$$F^{\mu\nu} = \pm\, {}^*F^{\mu\nu} \quad \text{at} \;\; \partial M$$

fixes coupling as $\gamma = \mp 1$

Quadratic-Curvature Couplings

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- **Examples in 3D: Topologically Massive Gravity (TMG), New Massive Gravity (NMG)**

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- **Arbitrary Quadratic Curvature Couplings**

$$I = \frac{1}{16\pi G} \int_M d^4x \sqrt{-g} \left(R - 2\Lambda + \alpha R^{\mu\nu} R_{\mu\nu} + \beta R^2 \right)$$

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- **Gauss-Bonnet term (topological invariant in 4D)**

$$GB = \sqrt{-g} \left(Rie^2 - 4R^{\mu\nu} R_{\mu\nu} + R^2 \right)$$

- **Equations of motion**

Classical Aspects of QCG

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$$E^{\mu\nu}_{(\beta)} = 2R \left(R^{\mu\nu} - \frac{1}{4} g^{\mu\nu} R \right) + 2 \left(g^{\mu\nu} \Box - \nabla^{\mu} \nabla^{\nu} \right) R.$$

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- **Vacua of the theory (maximally-symmetric spaces)**

$$R^{\mu\nu}_{\alpha\beta} = -\frac{1}{\ell^2_{\mathrm{eff}}}\,\delta^{\mu\nu}_{\alpha\beta}$$

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- β part

$$n_\mu \Theta^\mu_{(\beta)}(\delta g, \delta\Gamma) = \frac{\beta}{8\pi G}\delta^{[\mu\nu]}_{[\sigma\lambda]}\left[n_\mu R g^{\lambda\varepsilon}\delta\Gamma^\sigma_{\nu\varepsilon} - n^\lambda \nabla_\mu R \left(g^{-1}\delta g\right)^\sigma_\nu\right]$$

Abbott-Deser-Tekin energy

- **Perturbation around a background metric** $\bar{g}_{\mu\nu}$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

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- **Linearized curvatures**

$$R^{L}_{\mu\nu} = \frac{1}{2}\left(\bar{\nabla}_{\mu}\bar{\nabla}_{\nu} - \bar{\Box} h_{\mu\nu}\right)$$

$$R^{L} = \Lambda h$$

- **Conserved quantities**

$$
\begin{aligned}
8\pi G\, Q^{\mu}_{ADT}[\bar{\xi}] \;&=\; [1+2\Lambda\,(\alpha+4\beta)] \int\limits_{\partial M} d^3x\, G_L^{\mu\lambda}\bar{\xi}_\lambda + \\
&+ (\alpha+2\beta)\int\limits_{\Sigma} dS_\nu \left(2\bar{\xi}^{[\mu}\bar{\nabla}^{\nu]}R^L + R^L\bar{\nabla}^{\mu}\bar{\xi}^{\nu}\right) - \\
&-\alpha\int\limits_{\Sigma} dS_\nu \left(2\bar{\xi}_\lambda\bar{\nabla}^{[\mu}G_L^{\nu]\lambda} + 2G_L^{\lambda[\mu}\bar{\nabla}^{\nu]}\bar{\xi}_\lambda\right)\,.
\end{aligned}
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Abbott-Deser-Tekin energy and Criticality

- Schwarzschild-AdS black hole

$$M = m\left(1 + 2\Lambda(\alpha + 4\beta)\right).$$

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  [H. Lu and C.N. Pope, arXiv:1101.1971]

$$I_{critical} = \frac{1}{16\pi G}\int\limits_{M} d^4x\sqrt{-g}\left[R - 2\Lambda + \frac{3}{2\Lambda}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)\right]$$

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  -evaluation on particular black hole solutions

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- Linearization instability
  `[E. Altas and B. Tekin, arXiv:1705.10234]`

Noether-Wald charges

- **For a gravity Lagrangian** $L = L(g_{\mu\nu}, R_{\mu\nu\alpha\beta}, R_{\mu\nu}, R)$, the Noether current is

$$\sqrt{g}\,J^{\mu}[\xi] = \Theta^{\mu}(\delta_{\xi}g) + \Theta^{\mu}(\delta_{\xi}\Gamma) + L\xi^{\mu}\,.$$

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$$\sqrt{g}\, J^\mu = 2E^{\mu\nu\alpha\beta}\nabla_\nu \nabla_\alpha \xi_\beta ,$$

- **and the corresponding charge**

$$Q[\xi] = \int\limits_\Sigma d^2x\,\sqrt{\sigma}\, n_\mu u_\nu \nabla^\alpha \xi^\beta \frac{\delta L}{\delta R^{\alpha\beta}_{\mu\nu}} .$$

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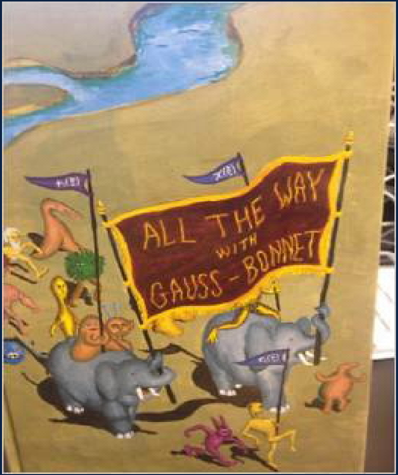
All the way with Gauss-Bonnet (Spivak)



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- **Einstein+Gauss-Bonnet in 4D ($\Lambda = -3/\ell^2$)**

$$I = \frac{1}{16\pi G} \int_M d^4x \sqrt{-g} \left[R - 2\Lambda + \gamma \left(R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} - 4 R^{\mu\nu} R_{\mu\nu} + R^2 \right) \right]$$

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- **GB is locally a surface term**

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- **Extrinsic counterterms (Kounterterms)**
  `R.O., [hep-th/0504233,hep-th/0610230]`

$$I = I_{EH} + \frac{\ell^2}{16\pi G}\int_{\partial M} d^3x\sqrt{-h}\,\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}K^{j_2}_{i_2}K^{j_3}_{i_3}\right).$$

Extrinsic Counterterms

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$$I = I_{EH} - \frac{1}{8\pi G} \int\limits_{\partial M} d^3x \sqrt{-h}\, K + \int\limits_{\partial M} d^3x\, \mathcal{L}_{ct}.$$

$$\mathcal{L}_{ct} = \frac{\ell^2}{16\pi G} \sqrt{-h} \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} + \frac{1}{\ell^2} \delta^{j_2}_{i_2} \delta^{j_3}_{i_3} \right)$$

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- Expansion of K^j_i for any AAdS spacetime

$$K^i_j = \frac{1}{\ell} \delta^i_j - \ell S^i_j(h) + \mathcal{O}(\mathcal{R}^2)$$

$$S^i_j(h) = \frac{1}{D-3} (\mathcal{R}^i_j(h) - \frac{1}{2(D-2)} \delta^i_j \mathcal{R}(h))$$

From Extrinsic to Intrinsic Counterterms

- O. Miskovic and R.O., [arXiv:0902.2082]

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`O.Miskovic and R.O., [arXiv:0902.2082]`

- **For Einstein spaces** ($B_{\mu\nu} = 0$), **CG action is equal to the renormalized Einstein action**

`G. Anastasiou and R.O., [arXiv:1608.07826]`

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  `L. Andrianopoli and R. D'Auria [arXiv:1405.2010]`
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