Near horizon of the OTT black hole, asymptotic symmetry and soft hair

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Outline

- Extremal black hole and near horizon
- Canonical Charges and asymptotic symmetry
- Poincare gauge theory of gravity in 3D
- Extremal static OTT
- Extremal rotating OTT
- Soft hair interpretation

Extremal black hole

Metric of the general stationary black hole

$$ds^2 = f(r)dt^2 - \frac{r^2}{g(r)} - d\Sigma^2(r) \tag{0.1}$$

Horizon is located at $r = r_0$ which satisfies

$$f(r_0) = g(r_0) = 0 \tag{0.2}$$

Extremal black hole has zero temperature $T = 0$ or equivalently

$$f'(r_0) = g'(r_0) = 0, \tag{0.3}$$

r_0 is a double zero.

Near horizon of extremal black hole

We zoom in the area of space-time near the horizon

$$r = r_0 + \alpha\epsilon\rho \tag{0.4}$$

$$t = \beta\frac{\tau}{\epsilon} \tag{0.5}$$

$$\epsilon \to 0 \tag{0.6}$$

after applying previous transformations and limes to the metric of extremal black hole we obtain

$$ds^2 = \frac{1}{2}\alpha^2\beta^2 f''(r_0)\rho^2 d\tau^2 - \frac{2d\rho^2}{g''(r_0)\rho^2} - d\Sigma^2(\lim_{\epsilon\to 0} r_0 + \alpha\epsilon\rho) \tag{0.7}$$

which is also the **solution** of the theory.

Canonical charges

Theory with local symmetry $\implies$ Presence of constraints $\mathcal{C}_i = 0$

Generator of local symmetries in $d+1$ dimensions is of the form

$$G = \int d^d x \alpha^i(x,t) C_i \tag{0.8}$$

Transformations are generated via Poison bracket

$$\delta A = [A, G] = \sum_{i,j} \frac{\delta A}{\delta \phi^i}\frac{\delta G}{\delta \pi_j} - \frac{\delta G}{\delta \phi^i}\frac{\delta A}{\delta \pi_j} \tag{0.9}$$

Canonical charges

Functional derivatives have to be well-defined

$$\delta G = \int d^d x \Phi_i \delta\phi^i + \Pi^i \pi_j + \int_{r\to\infty} Boundary\ \ term \tag{0.10}$$

Improved Generator

$$\tilde{G} = G + \Gamma \tag{0.11}$$

$$\Gamma = boundary\ \ term \tag{0.12}$$

Has well-defined functional derivatives

$$\delta\tilde{G} = \int d^d x \Phi_i \delta\phi^i + \Pi^i \pi_j \tag{0.13}$$

Integrability condition on charges

$$\delta\Gamma = -\int_{r\to\infty} Boundary\ \ term \tag{0.14}$$

Canonical charges and asymptotic symmetry

Surface term Γ represents **conserved charges**

If $\Gamma = 0 \implies$ symmetry is pure **gauge**

If $\Gamma \neq 0 \implies$ symmetry is **not gauge**

Boundary conditions are essential

Allowed symmetry=Local symmetries that respect boundary conditions

Asymptotic symmetry$=\frac{Allowed\ \ symmetry}{Symmetry\ \ with\ \ \Gamma=0}$

Poincare gauge theory in 3D

Dynamical variables

- vielbein $e^i{}_\mu$
- spin connection $\omega^i{}_\mu$

$$\omega^{ij}{}_\mu = -\epsilon^{ijk}\omega_{k\mu} \tag{0.15}$$

Field strengths

- Torsion $T^i = de^i + \omega^i{}_j e^j$
- Curvature $R^{ij} = d\omega^{ij} + \omega^i{}_k \omega^{kj}$

The most general parity preserving Lagrangian in 3D

$$\begin{aligned}
L_G &= -a_0 \varepsilon_{ijk} e^i R^{jk} - \frac{1}{3}\Lambda_0 \varepsilon_{ijk} e^i e^j e^k + L_{T^2} + L_{R^2}\,, \\
L_{T^2} &= T^{i\star}\left(a_1{}^{(1)}T_i + a_2{}^{(2)}T_i + a_3{}^{(3)}T_i\right)\,, \\
L_{R^2} &= \frac{1}{2}R^{ij\star}\left(b_4{}^{(1)}R_{ij} + b_5{}^{(5)}R_{ij} + b_6{}^{(6)}R_{ij}\right)\,.
\end{aligned} \tag{0.16}$$

Oliva-Tempo-Troncoso black hole is solution iff

$$b_4 + 2b_6 = 0\,. \tag{0.17}$$

Static Oliva-Tempo-Troncoso black hole

OTT black hole metric is given by

$$ds^2 = N^2 dt^2 - N^{-2} dr^2 - r^2 d\varphi^2 \,, \tag{0.18}$$

where $N^2 = -\mu + br + \dfrac{r^2}{\ell^2}$.
Horizons are located at

$$r_\pm = \frac{1}{2}\left(-b\ell^2 \pm \ell\sqrt{b^2\ell^2 + 4\mu}\right) \,. \tag{0.19}$$

The black hole is extremal iff horizons coincide $r_+ = r_-$. This condition is satisfied if

$$b^2\ell^2 + 4\mu = 0 \tag{0.20}$$

Near horizon of extremal static OTT

$$t \to \frac{t}{\varepsilon}\,, \qquad r \to r_+ + \varepsilon\rho\,. \tag{0.21}$$

After change of coordinates and limes $\varepsilon \to 0$ we obtain

$$ds^2 = \frac{r^2}{\ell^2}dt^2 - \frac{\ell^2}{r^2} - r_+^2 d\varphi^2\,. \tag{0.22}$$

$$e^0 = \frac{r}{\ell}dt\,, \qquad e^1 = \frac{\ell}{r}dr\,, \qquad e^2 = r_+ d\varphi\,. \tag{0.23}$$

Boundary conditions

We use notation

$$\mathcal{O}_n = \mathcal{O}(r^{-n}) \tag{0.24}$$

Asymptotic conditions on vielbein

$$e^i{}_\mu \sim \begin{pmatrix} \mathcal{O}_{-1} & \mathcal{O}_3 & \mathcal{O}_2 \\ \mathcal{O}_1 & \frac{\ell}{r} + \mathcal{O}_2 & \mathcal{O}_0 \\ \mathcal{O}_1 & \mathcal{O}_2 & \mathcal{O}_0 \end{pmatrix} \tag{0.25}$$

Asymptotic form of spin connection

$$\omega^i{}_\mu \sim \begin{pmatrix} \mathcal{O}_1 & \mathcal{O}_2 & \mathcal{O}_1 \\ \mathcal{O}_1 & \mathcal{O}_3 & \mathcal{O}_1 \\ \mathcal{O}_{-1} & \mathcal{O}_3 & \mathcal{O}_2 \end{pmatrix} \tag{0.26}$$

Asymptotic symmetry

The diffeomorphisms

$$\begin{aligned} \xi^t &= T(t) + \mathcal{O}_3\,, \\ \xi^r &= rU(\varphi) + \mathcal{O}_0\,, \\ \xi^\varphi &= S(\varphi) + \mathcal{O}_1\,. \end{aligned} \tag{0.27}$$

Local Loretz transformations

$$\begin{aligned} \theta^0 &= \mathcal{O}_2\,, \qquad \theta^1 = \mathcal{O}_2 \\ \theta^2 &= \mathcal{O}_2\,. \end{aligned} \tag{0.28}$$

All surface terms are zero $\Gamma = 0$

Symmetry is pure gauge

Rotating OTT black hole

Metric of the rotating OTT

$$ds^2 = N^2 dt^2 - F^{-2} dr^2 - r^2 (d\varphi + N_\varphi dt)^2 \,, \qquad (0.29a)$$

where

$$F = \frac{H}{r}\sqrt{\frac{H^2}{\ell^2} + \frac{b}{2} H \left(1+\eta\right) + \frac{b^2 \ell^2}{16} \left(1-\eta\right)^2 - \mu\eta} \,,$$

$$N = AF \,, \qquad A = 1 + \frac{b\ell^2}{4H}(1-\eta) \,,$$

$$N_\varphi = \frac{\ell}{2r^2}\sqrt{1-\eta^2}(\mu - bH) \,,$$

$$H = \sqrt{r^2 - \frac{\mu\ell^2}{2}(1-\eta) - \frac{b^2\ell^4}{16}\left(1-\eta\right)^2} \,. \qquad (0.29b)$$

Near horizon of rotating OTT

There are two possible near horizon geometries

- $4\mu + b^2 l^2 = 0$ leads to the previous case
- $\eta = 0$

$$ds^2 = \frac{2r^2 r_0}{\ell^2} dt d\varphi - \frac{\ell^2}{r^2} dr^2 - r_0^2 d\varphi^2, \tag{0.30}$$

We chose triads to be of the form

$$e^0 = \frac{r^2}{\ell^2} dt\,, \qquad e^1 = \frac{\ell}{r} dr\,, \qquad e^2 = \frac{r^2}{\ell^2} dt - r_0 d\varphi\,. \tag{0.31}$$

Boundary conditions

Asymptotic form of vielbein

$$e^{i}{}_{\mu} \sim \begin{pmatrix} \frac{r^2}{\ell^2} + \mathcal{O}_1 & \mathcal{O}_5 & \mathcal{O}_0 \\ \mathcal{O}_1 & \dfrac{\ell}{r} + \mathcal{O}_3 & \mathcal{O}_0 \\ \frac{r^2}{\ell^2} + \mathcal{O}_1 & \mathcal{O}_5 & \mathcal{O}_0 \end{pmatrix} \tag{0.32}$$

Asymptotic form of the spin connection

$$\omega^{i}{}_{\mu} \sim \begin{pmatrix} -\frac{r^2}{\ell^3} + \mathcal{O}_1 & \mathcal{O}_2 & \mathcal{O}_0 \\ \mathcal{O}_0 & -\frac{1}{r} + \mathcal{O}_2 & \mathcal{O}_0 \\ -\frac{r^2}{\ell^3} + \mathcal{O}_1 & \mathcal{O}_2 & \mathcal{O}_0 \end{pmatrix} \tag{0.33}$$

Asymptotic symmetry

The diffeomorphisms

$$\xi^t = T(t) + \mathcal{O}_3\,,$$
$$\xi^r = rU(\varphi) + \mathcal{O}_1\,,$$
$$\xi^\varphi = S(\varphi) + \mathcal{O}_4\,. \tag{0.34}$$

Loretz transformations

$$\theta^2 = \mathcal{O}_1 \tag{0.35}$$
$$\theta^1 = -\frac{2\xi^r}{r} - \partial_t \xi^t + \mathcal{O}_4$$
$$\theta^0 = \mathcal{O}_1.$$

Asymptotic symmetry: Charges

$$\tilde{G} = G + \Gamma \tag{0.36}$$

$$\Gamma = -4a_0 \int_0^{2\pi} d\varphi [T(t)\frac{r^2}{\ell^2}(\omega_\varphi^0 - \frac{e_\varphi^0}{\ell} - \omega_\varphi^2 + \frac{e_\varphi^2}{\ell}) + \tag{0.37}$$

$$S(\varphi)\omega_\varphi^i e_{i\varphi} + (2U(\varphi) + \partial_t T(t))e_\varphi^1] \tag{0.38}$$

The charge is finite due to conditions that follow from the constraint $T^i = 0$

$$\frac{e^0{}_\varphi}{\ell} - \frac{e^2{}_\varphi}{\ell} + \omega_\varphi^2 - \omega_\varphi^0 = \mathcal{O}_2. \tag{0.39}$$

Asymptotic symmetry: Algebra

Using the composition law for the local Poincare transformations

$$\xi''^{\mu} = \xi^{\alpha}\partial_{\alpha}\xi'^{\mu} - \xi'^{\alpha}\partial_{\alpha}\xi^{\mu} \tag{0.40}$$

$$\theta''^{i} = \epsilon^{i}{}_{jk}\theta^{j}\theta'^{k} + \xi^{\alpha}\partial_{\alpha}\theta'^{i} - \xi'^{\alpha}\partial_{\alpha}\theta^{i} \tag{0.41}$$

we obtain the algebra of charges Virasoro algebra

$$[L_m, L_n] = -i(m-n)L_{m+n}. \tag{0.42}$$

Kac-Moody algebra

$$[J_m, J_n] = -i16\pi a_0 \ell m \delta_{m+n,0}, \tag{0.43}$$

where

$$\kappa = 16\pi a_0 \ell = \frac{\ell}{G}. \tag{0.44}$$

Entropy

Entropy of the extremal OTT black hole

$$S = \pi \frac{r_0}{G} \tag{0.45}$$

can be reproduced via peculiar formula

$$S = 4\pi \sqrt{\frac{1}{8} L_0^{on-shell} \kappa}, \tag{0.46}$$

where the value of the Virasoro generator L_0 on the solution

$$L_0^{on-shell} = \frac{r_0^2}{2\ell G}. \tag{0.47}$$

Entropy is reproduced with entropy formula of WCFT

$$S_{WCFT} = -\frac{i2\pi J_0 J_0^{vac}}{\kappa} + 4\pi \sqrt{-(L_0 - \frac{(J_0)^2}{2\kappa})(L_0^{vac} - \frac{(J_0^{vac})^2}{2\kappa})} \tag{0.48}$$

IFF

$$L_0^{vac} - \frac{(J_0^{vac})^2}{2\kappa} = -\frac{\kappa}{8} \tag{0.49}$$

Soft Hair

Hair $\iff$ difference between states of a black hole

Soft $\iff$ zero energy

Near Horizon charges are soft

∂_τ is Killing vector

near horizon states with energy ω are of the form

$$e^{i\omega\tau} \tag{0.50}$$

From the perspective of Black hole space-time

$$e^{i\omega\tau} = e^{i\omega\epsilon t} \tag{0.51}$$

consequently, the energy is

$$\Omega = \omega\epsilon \to 0 \tag{0.52}$$

Conclusion

Soft hair is promising proposal for the explanation of the black hole entropy and information paradox

Simple realization of soft hair for extremal black holes via near horizon geometry

Still far from full picture, many open problems

Thank you for your attention