Symmetries of the gauge theories in the light front

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arXiv: 2302.xxxxx

Workshop on Gravity, Holography, Strings & Noncommutative Geometry

3rd February 2023, Belgrade, Serbia

Outline

1. Infrared structure of gauge theories

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2. Hamiltonian analysis of electromagnetism in the null foliation

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3. New symmetry generator; Beyond $\mathrm{U}(1)$

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3. New symmetry generator; Beyond $\mathrm{U}(1)$
4. Extension to Yang-Mills theory
5. Discussion

Infrared structure of gauge theories

IR region of theories with massless particles in asymptotically flat spaces

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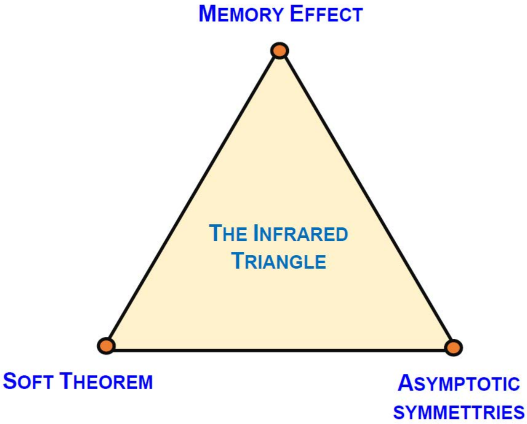

Infrared structure of gauge theories

Motivation

- **Hamiltonian treatment of asymptotic symmetries**

  [Bondi, van der Burg, Metzner 1962; Sachs 1962]

- **BMS symmetry** – infinite-dimensional asymptotic symmetry at the null boundary of 4D asymptotically flat spacetimes

- **Celestial holography**

- Duality between massless 4D asymptotically flat spacetimes and 2D CFT on the celestial sphere

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- **Asymptotic symmetries in Electromagnetic and Yang-Mills theories**

- 2D realization of soft symmetries in electromagnetism

  [He, Mitra, Porfyriadis, Strominger 2014; Nande, Pate, Strominger 2018]

- Extension to Yang-Mills [Strominger 2014; He, Mitra, Strominger, 2016]

Infrared structure of gauge theories

Vacuum degeneracy in gauge theories $(\omega \longrightarrow 0)$

$\Leftrightarrow$ Enhancement of symmetries at the boundary of flat spacetime $(r \longrightarrow \infty)$

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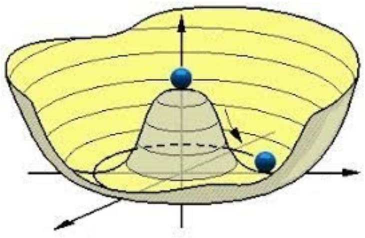
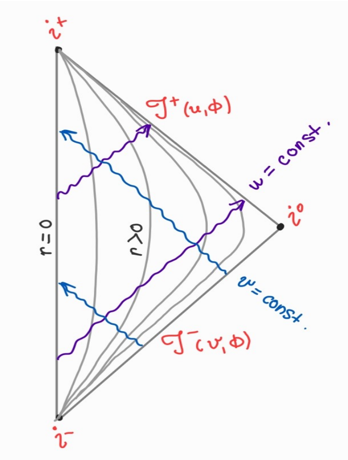


Penrose diagram of the Minkowski space



Hamiltonian analysis of electromagnetism in the null foliation

Null foliated reference frame

- **Minkowski metric in $D = 4$ in the spherical coordinates** (t, r, y^A)

$$M_4 : \quad \mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2$$

$$\mathbb{S}^2 : \quad \mathrm{d}\Omega^2 = \gamma_{AB}(y)\, \mathrm{d}y^A \mathrm{d}y^B$$

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- **Time coordinate** $\quad u = t - \epsilon r \,, \quad -1 \leq \epsilon \leq 1$

$\epsilon = 1$	retarded time
$\epsilon = 0$	proper time of a massive particle
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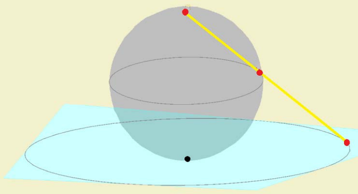
- **Coordinates on $\mathbb{S}^2$: stereographic projection** $(\theta, \varphi) \to y^A = (z, \bar{z})$

$$
z = \mathrm{e}^{\mathrm{i}\varphi} \cot \tfrac{\theta}{2}\,, \quad \bar{z} = \mathrm{e}^{-\mathrm{i}\varphi} \cot \tfrac{\theta}{2}
$$

Hamiltonian analysis of electromagnetism in the null foliation



- **Minkowski metric $\mathfrak{g}_{\mu\nu}$ in the coordinates $x^\mu = (u, r, y^A)$:**

$$\boxed{\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\epsilon\,\mathrm{d}u\mathrm{d}r + \left(1-\epsilon^2\right)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2}$$

- **$\mathbb{S}^2$ metric in complex coordinates**

$$\gamma_{AB} = \begin{pmatrix} 0 & \gamma_{z\bar{z}} \\ \gamma_{z\bar{z}} & 0 \end{pmatrix}, \quad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2} = \sqrt{\gamma}$$

Hamiltonian analysis of electromagnetism in the null foliation

- **Electromagnetic action in the background** $\mathfrak{g}_{\mu\nu}$

$$I[A] = -\frac{1}{4e^2}\int \mathrm{d}^4x\,\sqrt{\mathfrak{g}}\,F^{\mu\nu}F_{\mu\nu} \qquad (F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$$

- **Canonical momenta** $\pi^\mu = -\frac{1}{e^2}\sqrt{\mathfrak{g}}\,F^{u\mu}$

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In components:

$$\boxed{\begin{aligned} \pi^u &= 0 \\ \pi^r &= \frac{r^2}{e^2}\sqrt{\gamma}\,F_{ur} \\ \pi^A &= -\frac{1}{e^2}\sqrt{\gamma}\gamma^{AB}\left[(\epsilon^2-1)F_{uB} - \epsilon F_{rB}\right] \end{aligned}}$$

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- The limit $\epsilon^2 \to 1$ is discontinuous
- The action in the light-cone ($\epsilon^2 = 1$) has an additional constraint
- Light-cone actions are first order in velocities [Steinhardt 1980]

Hamiltonian analysis of electromagnetism in the null foliation

Bondi reference frame: $\epsilon^2 = 1$

- **Primary constraints** $\boxed{\pi^u \approx 0\,, \quad \chi^A \equiv \epsilon\pi^A - \frac{1}{e^2}\sqrt{\gamma}\gamma^{AB}F_{rB} \approx 0}$

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- **Total Hamiltonian** [Dirac 1964]

$$\mathcal{H}_T = \mathcal{H}_C + \lambda_u\pi^u + \lambda_A\chi^A$$

$\lambda_u(x)$, $\lambda_A(x) =$ Hamiltonian multipliers

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- **Canonical Poisson brackets** $\{A_\mu(x), \pi^\nu(x')\}_{u=u'} = \delta_\mu^\nu\,\delta^{(3)}(x-x')$

- **Evolution** $\dot{\Phi}(x) \approx \{\Phi(x), H_T\} = \int \mathrm{d}^3x'\,\{\Phi(x), \mathcal{H}_T(x')\}$

Hamiltonian analysis of electromagnetism in the null foliation

Comment

- We always have two different Hamiltonians $\epsilon = \pm 1$.
- The future cone $\epsilon = +1$ and the past cone $\epsilon = -1$ are causally disconnected.
- Common boundary i^0 is located at the spatial infinity.

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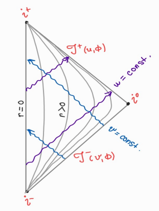
- Common boundary i^0 is located at the spatial infinity.
- A radial wave located at (u, r) propagates along $v = t + \epsilon r = const.$
- In absence of massive particles, the future lightlike infinity $\mathcal{J}^+ = \mathcal{J}^+(t - r, y)$ and the past lightlike infinity $\mathcal{J}^- = \mathcal{J}^-(t + r, y)$ behave as Cauchy surfaces [Hawking, Perry, Strominger 2016].



Hamiltonian analysis of electromagnetism in the null foliation

Symplectic matrix

$$\boxed{\left\{\chi^A(x),\chi^B(x')\right\} = \Omega^{AB}(x,x') \equiv -\frac{2\epsilon}{e^2}\sqrt{\gamma}\gamma^{AB}\partial_r\delta^{(3)}}$$

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- If Ω^{AB} - not invertible: χ^A are **second class** (eliminate redundant fields)

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- Ω^{AB} is invertible because γ^{AB} is invertible $\Rightarrow$ χ^A are second class
- Reduced phase space $\chi^A = 0$ [Goldberg 1991, Majumdar 2022]

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Other possibility

- Ω^{AB} is invertible, but its inverse is not unique
- Ω^{AB} is infinite-dimensional matrix and it has zero modes

$$\int d^3x'\,\Omega^{AB}V'_B = -\tfrac{2\epsilon}{e^2}\sqrt{\gamma}\gamma^{AB}\partial_r V_B = 0 \quad \Rightarrow \quad V_B = V_B(y)$$

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$\Rightarrow \chi^A_{(0)}(y)$ **is first class constraint** (r-independent part of the constraint)

Hamiltonian analysis of electromagnetism in the null foliation

Consistency conditions

Hamiltonian analysis of electromagnetism in the null foliation

Consistency conditions

- **Conservation of constraints during their evolution**

$$\dot{\pi}^u = 0 \qquad \Rightarrow \quad \chi = \partial_i \pi^i \approx 0 \quad \text{(differential Gauss law)}$$

$$\dot{\chi}^A = 0 \qquad \Rightarrow \quad \int \mathrm{d}^3x'\, \Omega^{AB} \lambda'_B = \int \mathrm{d}^3x'\, \{\mathcal{H}'_C, \chi^A\}$$

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- **The multiplier λ_A is not fully determined**

$$\partial_r \lambda_A = -\frac{\epsilon e^2}{2\sqrt{\gamma}} \partial_r \tilde{\pi}_A - \frac{1}{2r^2} \nabla^B F_{AB} + \frac{\epsilon e^2}{2r^2} \partial_B \left(\frac{\pi^r}{\sqrt{\gamma}} \right)$$

$$\lambda_A = \bar{\lambda}_A + \Lambda_A(y)$$

- A free function $\Lambda_A(y)$ is due to the zero modes of Ω^{AB}
- $\bar{\lambda}_A$ - determined part of λ_A

Hamiltonian analysis of electromagnetism in the null foliation

Summary of the constraints

Primary constraints: $\pi^u\,,\quad \chi^A = \epsilon\pi^A - \frac{1}{e^2}\sqrt{\gamma}\gamma^{AB}F_{rB}$

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- $\chi^A_{(0)}$ – first class, r-independent part of the constraint
- $\chi^A_{(n)}$ $(n \geq 1)$ – second class, coefficients of the Taylor expansion in $\frac{1}{r}$
- We have to expand all the fields asymptotically in the vicinity of the boundary $r = const \longrightarrow \infty$.

Hamiltonian analysis of electromagnetism in the null foliation

Standard asymptotic conditions of the fields [Strominger 2014]

$$\boxed{\begin{aligned} A_u &= \mathcal{O}(\tfrac{1}{r})\,, & A_r &= \mathcal{O}(\tfrac{1}{r^2})\,, & A_A &= \mathcal{O}(r^0)\,, \\ \pi^u &= 0\,, & \pi^r &= \mathcal{O}\left(r^0\right)\,, & \pi^A &= \mathcal{O}(\tfrac{1}{r^2}) \end{aligned}}$$

- **Boundary fields**: $A_{(0)A} = A_A|_{r\to\infty}$, $\quad \pi^r_{(0)} = \pi^r|_{r\to\infty}$

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- **Boundary fields**: $A_{(0)A} = A_A|_{r\to\infty}, \quad \pi^r_{(0)} = \pi^r|_{r\to\infty}$
- **Extended Hamiltonian** $\quad \mathcal{H}_E = \mathcal{H}_C + \lambda_u \pi^u + \lambda_A \chi^A + \lambda\, \partial_i \pi^i$
- **Hamilton's equations**

$$\boxed{\begin{aligned} \dot{A}_u &= \lambda_u\,, & \dot{A}_r &= \frac{e^2 \pi^r}{r^2\sqrt{\gamma}} - \partial_r \lambda\,, & \dot{A}_A &= \frac{e^2\,\tilde{\pi}_A}{\sqrt{\gamma}} - \partial_A \lambda + \epsilon \lambda_A \\ \dot{\pi}^u &= 0\,, & \dot{\pi}^r &= \frac{\sqrt{\gamma}}{e^2}\,\nabla_A \lambda^A\,, & \dot{\pi}^A &= -\frac{\sqrt{\gamma}}{e^2}\left(\frac{1}{r^2}\,\nabla_B \tilde{F}^{AB} + \partial_r \lambda^A\right) \end{aligned}}$$

- **Fall-off of the multipliers**

$$\lambda_u = \mathcal{O}(\tfrac{1}{r})\,, \qquad \Lambda_A = \mathcal{O}\left(r^0\right)\,, \qquad \lambda = \mathcal{O}(\tfrac{1}{r})$$

A new symmetry generator

SUMMARY

1$^{\text{st}}$ class constr.	Multipliers	Parameters	Generators	Charges
$\pi^u,\ \chi = \partial_i \pi^i$	$\lambda_u,\ \lambda$	$\theta_u,\ \theta$	$G[\theta]$	$Q[\theta]$
$\chi^A_{(0)}$	$\Lambda_A(\phi)$	$\eta_A(\phi)$	$S[\eta]$	$Q_s[\eta]$

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- **Smeared generators**

$$
\begin{aligned}
G[\theta] &= \int \mathrm{d}^3x \ \left(\theta\, \partial_i \pi^i + \theta_u \pi^u\right) && \mathrm{U}(1) \text{ symmetry} \\
S[\eta] &= \int \mathrm{d}^3x \, \eta_A \chi^A && \text{asymptotic symmetry}
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- **New symmetry generator**

- χ^A contains $\chi^A_{(0)}$ (first class) and $\chi^A_{(n)}$ $(n \geq 1)$ (second class)

- The basis $\{r^n | n \in \mathbb{Z}\}$ is not complete, $\chi^A \to \chi^A_{(n)}$ not invertible

- We have to ensure that $\chi^A_{(n)}$ $(n \geq 1)$ do not contribute to δ_η

- η_A tends fast to $\eta_{(0)A}$, we can take $\eta_A = \eta_{(0)A}(y)$

A new symmetry generator

- **Transformation laws of the fields** $\delta_\theta = \{\ \ , G[\theta]\}$, $\delta_\eta = \{\ \ , S[\eta]\}$

$$\boxed{\begin{aligned} \delta_\theta A_\mu &= \theta_u\, \delta^u_\mu - \partial_i \theta\, \delta^i_\mu\,, & \delta_\eta A_\mu &= \epsilon\, \eta_A\, \delta^A_\mu \\ \delta_\theta \pi^\mu &= 0\,, & \delta_\eta \pi^\mu &= \tfrac{1}{e^2}\, \delta^\mu_r \sqrt{\gamma}\, \nabla_A \eta^A \end{aligned}}$$

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★ θ : Standard U(1) gauge transformations $\delta_\theta A_\mu = -\partial_\mu\theta$

★ η^A : Act only on the boundary fields (Goldstone modes)

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A new symmetry generator

- **Transformation laws of the fields** $\delta_\theta = \{\ \ , G[\theta]\}$, $\delta_\eta = \{\ \ , S[\eta]\}$

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- **Transformation law of the multipliers**

$$\delta\lambda_u = \dot\theta_u\,, \qquad \delta\lambda = \dot\theta + \mathcal{O}(\tfrac{1}{r})\,, \qquad \delta\Lambda_A = \epsilon\, \dot\eta_A + \mathcal{O}(\tfrac{1}{r})$$

- **Fall-off of the local parameters**

$$\theta_u = \mathcal{O}(\tfrac{1}{r})\,, \qquad \theta = \mathcal{O}(r^0)\,, \qquad \eta^A = \mathcal{O}(r^0)$$

- **Improper transformations**: $\theta_{(0)}$, $\eta^A_{(0)}$

[Benguria, Cordero, Teitelboim 1977]

A new symmetry generator

Improved generators and charges

- **Improved generators**

$$G_Q[\theta] = G[\theta] + Q[\theta] \quad (Q[\theta] = \text{surface term})$$
$$S_Q[\eta] = S[\eta] + Q_s[\eta] \quad (Q_s[\theta] = \text{surface term})$$

- **Differentiability**

$$\delta G_Q[\theta] = \int \mathrm{d}^3x \left(\frac{G_Q[\theta]}{\delta A_\mu} \delta A_\mu + \frac{G_Q[\theta]}{\delta \pi^\mu} \delta \pi^\mu \right)$$
$$\delta S_Q[\eta] = \int \mathrm{d}^3x \left(\frac{S_Q[\eta]}{\delta A_\mu} \delta A_\mu + \frac{S_Q[\eta]}{\delta \pi^\mu} \delta \pi^\mu \right)$$

A new symmetry generator

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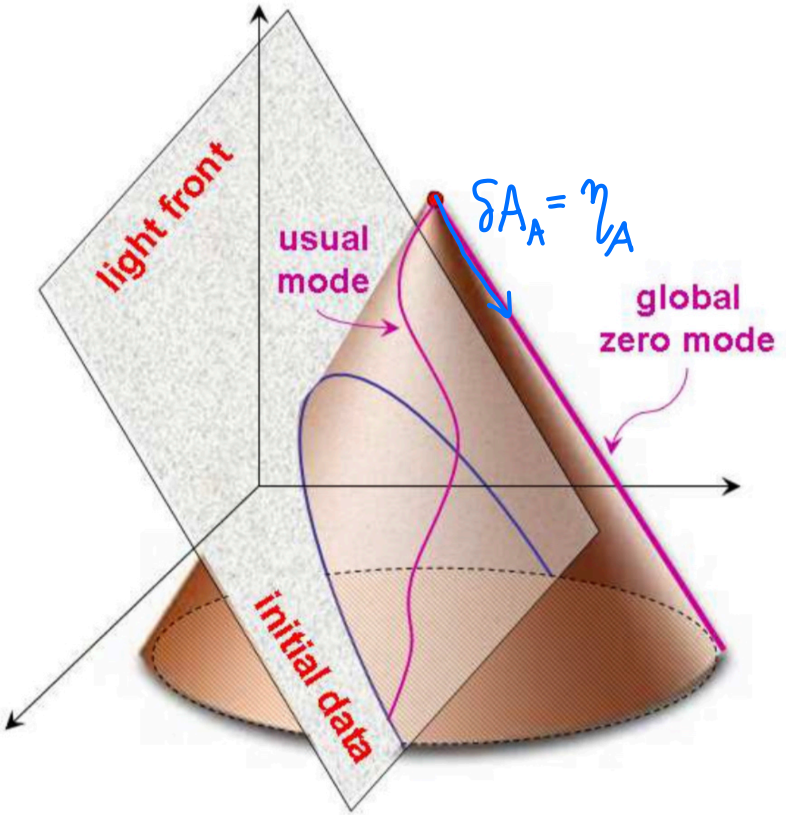
- **Charges** (field-independent local parameters)

$$Q[\theta] = -\oint \mathrm{d}^2y\, \theta\, \pi^r$$

$$Q_s[\eta] = \frac{1}{e^2} \oint \mathrm{d}^2y \sqrt{\gamma}\, \eta^A A_A$$

- The integral is over the two-sphere at the infinity.
- There is an infinite number of global charges, corresponding to the spherical harmonics mode expansion (or Laurent expansion) of the local parameters.

WHAT'S GOING ON?



[ALEXANDROV, SPEZIALE 2014]

Evolution is insensitive to the dynamics that is not prescribed on the initial data surface.

A new symmetry generator

Charge algebra

- Reduced phase space: $G_Q[\theta] = 0 + Q[\theta]$, $S_Q[\eta] = 0 + Q_s[\eta]$
- Algebra can be computed from $\delta_{\theta_2} Q[\theta_1] = \{Q[\theta_1], Q[\theta_2]\}$
- General form of the charge algebra: $\{Q[\theta_1], Q[\theta_2]\} = Q[[\theta_1, \theta_2]] + C[\theta_1, \theta_2]$
- **In the electromagnetic theory**

$$\boxed{\begin{aligned} \{Q[\theta_1], Q[\theta_2]\} &= 0 \\ \{Q_s[\eta_1], Q_s[\eta_2]\} &= 0 \\ \{Q[\theta], Q_s[\eta]\} &= C[\theta, \eta] \neq 0 \end{aligned}}$$

- **Central charge** $C[\theta, \eta] = \frac{1}{e^2} \oint \mathrm{d}^2 y \, \sqrt{\gamma}\, \eta^A \partial_A \theta$

A new symmetry generator

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- **Central charge** $C[\theta, \eta] = \frac{1}{e^2} \oint \mathrm{d}^2 y \sqrt{\gamma}\, \eta^A \partial_A \theta$
- **Holographic conjugate pairs on** $\mathbb{S}^2$ [Donnay,Puhm, Strominger 2019]

  $\{Q[\theta], Q_s[\eta]\} = C[\theta, \eta] \quad \leftrightarrow \quad \{q, p\} = 1$

  $Q[\theta]$ – conformally soft photon mode

  $Q_s[\eta]$ – Goldstone current

A new symmetry generator

Algebra in modes

- **Laurent series**

$$\psi(z,\bar{z}) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \frac{\psi_{nm}}{z^{n+h}\bar{z}^{m+\bar{h}}}$$

- The powers $(h,\bar{h})$ are related to the spin of the tensor ψ

- Scalars $\pi^r : (0,0)$

- Vectors:$A_z : (1,0)$, $A_{\bar{z}} : (0,1)$

A new symmetry generator

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- **Charges**

$$\begin{aligned} Q[\theta] &= \textstyle\sum_{n,m} \theta_{nm} G_{nm} \in \mathbb{R} \\ Q_s[\eta] &= \textstyle\sum_{n,m} (\eta_{nm}\bar{S}_{nm} + \bar{\eta}_{nm} S_{nm}) \in \mathbb{R} \end{aligned}$$

- **Generators**

$$\begin{aligned} G_{nm} &= 4\pi^2\, \pi_{1-n,1-m} \\ S_{nm} &= -\tfrac{4\pi^2}{e^2}\, A_{-n,-m} \\ \bar{S}_{nm} &= -\tfrac{4\pi^2}{e^2}\, \bar{A}_{-n,-m} \end{aligned}$$

A new symmetry generator

- **Algebra** (non vanishing brackets only)

$$\boxed{\begin{aligned} \{G_{nm}, S_{kl}\} &= \kappa n\, \delta_{n+k,0}\delta_{m+l,0} \\ \{G_{nm}, \bar{S}_{kl}\} &= \kappa m\, \delta_{n+k,0}\delta_{m+l,0} \end{aligned}}$$

- **Level of the algebra:** $\kappa = \frac{4\pi^2}{e^2}$

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- **Level of the algebra:** $\kappa = \frac{4\pi^2}{e^2}$
- **Change of the basis**: $(G_{nm}, S_{nm}, \bar{S}_{nm}) \to (R_{nm}, J_{nm}, \bar{J}_{nm})$
- **Generalization of Kac-Moody algebra**

$$\begin{aligned}\{J_{nm}, J_{kl}\} &= \kappa\,(n-m)\ \delta_{n+k,0}\delta_{m+l,0} \\ \{\bar{J}_{nm}, \bar{J}_{kl}\} &= -\kappa\,(n-m)\ \delta_{n+k,0}\delta_{m+l,0} \\ \{R_{nm}, J_{kl}\} &= \kappa n\, \delta_{n+k,0}\delta_{m+l,0} \\ \{R_{nm}, \bar{J}_{kl}\} &= \kappa\, m\, \delta_{n+k,0}\delta_{m+l,0} \\ \{R_{nm}, R_{kl}\} &= \kappa\,(n+m)\, \delta_{n+k,0}\delta_{m+l,0}\end{aligned}$$

A new symmetry generator

Abelian Kac-Moody subalgebras

- **We obtain six Abelian KM algebras** $\{j_n, j_m\} = \kappa n\, \delta_{n+m,0}$

Currents j_n	Levels
J_{n0}, J_{0n}	$\kappa, -\kappa$
$\bar{J}_{n0}, \bar{J}_{0n}$	$-\kappa, \kappa$
R_{n0}, R_{0n}	κ, κ

- **Non vanishing mixed brackets**: $\{R_{n0}, J_{m0}\}, \{R_{0n}, \bar{J}_{0m}\} \neq 0$
- Each KM algebra is naturally generated by current proportional to a holomorphic or anti-holomorphic functions.

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- Each KM algebra is naturally generated by current proportional to a holomorphic or anti-holomorphic functions.
- $\{J_{00}, \bar{J}_{00}, R_{00}\}$ span the global Abelian algebra

A new symmetry generator

Beyond $\mathrm{U}(1)$ – conformal symmetry

- Conformal plane – a realization of conformal symmetry described by Virasoro algebra
- Witt algebra – obtained from KM algebra using the Sugawara construction [Sugawara 1967]

A new symmetry generator

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- We have several Witt algebras, not all of them independent
- **Six Witt generators** $L_n = \frac{1}{2\kappa} \sum_k j_k j_{n-k}$
- **Six Witt algebras** $\{L_n, L_m\} = (n-m)\, L_{n+m}$
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- Quantization introduces a central extension.
- *In progress*: Relation to the global 4D Poincaré generators.

Extension to Yang-Mills theory

- **Zero mode**

$$\partial_r V_A = -[A_r, V_A] \quad \Rightarrow \quad V_A(x) = U^{-1} V_{(0)A}(y)\, U,$$

with $U = \exp\left(-\int_r^\infty \mathrm{d}r\, A_r\right)$ and the bdy. condition $V_A|_{r\to\infty} = V_{(0)A}(y)$

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- **Charges**

$$Q[\theta] = -\oint \mathrm{d}^2y\, \theta^a \pi^r_a\,, \quad Q_s[\eta] = \frac{1}{g^2}\oint \mathrm{d}^2y\, \sqrt{\gamma}\, \eta^a_A A^A_a$$

- **Local transformations**

$$\begin{aligned} \delta_{\theta,\eta} A^a_u &= \theta^a_u \qquad \delta_{\theta,\eta} A^a_r = -D_r\theta^a \\ \delta_{\theta,\eta} A^a_A &= -D_A\theta^a + \epsilon\, \eta^a_A \end{aligned}$$

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- **Non-Abelian charge algebra**

$$\boxed{\begin{aligned} \{Q[\theta_1], Q[\theta_2]\} &= Q[[\theta_1, \theta_2]] \\ \{Q[\theta], Q_s[\eta]\} &= Q_s[[\theta, \eta]] + \frac{1}{g^2}\oint \mathrm{d}^2y\, \sqrt{\gamma}\, \eta_a^A \partial_A \theta^a \\ \{Q_s[\eta_1], Q_s[\eta_2]\} &= 0 \qquad \longrightarrow Q_s \text{ is Abelian} \end{aligned}}$$

Extension to Yang-Mills theory

Yang-Mills theory

$$I[A] = -\frac{1}{4g^2}\int \mathrm{d}^4x\sqrt{\mathfrak{g}}\, F_a^{\mu\nu} F^a_{\mu\nu}; \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^a{}_{bc} A^b_\mu A^c_\nu$$

- **Constraints**

$$\boxed{\pi^u_a \approx 0\,, \quad \chi^A_a \equiv \epsilon\, \pi^A_a - \frac{1}{g^2}\sqrt{\gamma}\gamma^{AB} F^a_{rB} \approx 0, \quad \chi_a \equiv D_i \pi^i_a \approx 0}$$

Extension to Yang-Mills theory

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- **Constraint algebra**

$$\begin{aligned}
\{\chi_a, \chi'_b\} &= f_{ab}{}^c\,\chi_c\,\delta^{(3)} \\
\left\{\chi_a, \chi'^A_b\right\} &= f_{ab}{}^c\,\chi^A_c\,\delta^{(3)} \\
\left\{\chi^A_a, \chi'^B_b\right\} &= \Omega^{AB}_{ab}(x, x')
\end{aligned}$$

- **Non-Abelian symplectic matrix**

$$\Omega^{AB}_{ab}(x, x') = -\frac{2\epsilon}{g^2}\sqrt{\gamma}\gamma^{AB}\left(g_{ab}\,\partial_r + f_{abc}\,A^c_r\right)\delta^{(3)}$$

Extension to Yang-Mills theory

- **Mode algebra**

$$\boxed{\begin{aligned} \left\{G^a_{nm}, G^b_{kl}\right\} &= f^{ab}_{\ \ c}\, G^c_{n+k,m+l} \\ \left\{G^a_{nm}, S^b_{kl}\right\} &= f^{ab}_{\ \ c}\, S^c_{n+k,m+l} + \kappa n g^{ab}\delta_{n+k,0}\delta_{m+l,0} \\ \left\{G^a_{nm}, \bar{S}^b_{kl}\right\} &= f^{ab}_{\ \ c}\, \bar{S}^c_{n+k,m+l} + \kappa m g^{ab}\delta_{n+k,0}\delta_{m+l,0} \end{aligned}}$$

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- **Level** $\kappa = \frac{4\pi^2}{g^2}$
- One can apply the Sugawara method again...
- *We conclude that the symmetries at the asymptotic null boundary, described by KM algebras and Virasoro algebras, are general features of 4D gauge theories*

Discussion

Asymptotic conditions

- Invariance of boundary conditions under Poincaré transformatons is not straighforward
- Hamiltonian treatment at spatial infinity needs additional **parity conditions** to ensure invariance under boosts.

- Electromagnetism [Henneaux, Troessaert 2018] ;
- Yang-Mills [Tanzi, Giulini 2020]

- Null-slices foliated standard b.c. in electromagnetism are invariant under Poincaré group. [Bunster, Gomberoff, Pérez 2018]
- We showed the Poincaré invariance in the non-Abelian case.

Discussion

Poincaré transfromations

- We found several Kac-Moody algebras, but not all of them are related to the global Poincaré symmetry in 4D spacetime.
- We constructed a generator of 4D Poincaré transformations and its action at the light front, by writing the YM stress tensor in the canonical form,

$$T^{\mu}{}_{\nu} = \frac{1}{g^2} \left(F_a^{\mu\alpha} F^a_{\nu\alpha} - \frac{1}{4}\, \delta^{\mu}_{\nu}\, F_a^{\alpha\beta} F^a_{\alpha\beta} \right)$$

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To be done

- Hamiltonian treatment of the gravitatonal action using the null foliation.
- Description of a holographic theory.
- Addition of the θ-term in the action (Pontryagin topological invariant with the couplig θ), which will change the central charges.

Acknowledgments

THANK YOU!

Acknowledgments

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Agencia Nacional de Investigación y Desarrollo
Ministerio de Ciencia, Tecnología, Conocimiento e Innovación
Gobierno de Chile

Fondecyt
Fondo Nacional de Desarrollo Científico y Tecnológico

Acknowledgments

FONDECYT Grant N° 1190533
Anillo Grant ANID/ACT210100
Holography and its applications to high energy physics, quantum gravity and condensed matter systems

FONDECYT Grant No. 1190533
Black holes and asymptotic symmetries