Conformal Renormalization and Energy Functionals in AdS gravity

2023 Workshop on Gravity, Holography, Strings and Noncommutative Geometry
Feb 3th, 2023, Belgrade

Rodrigo Olea

UNAB, Chile

HOLOGRAPHYCL

Outline

Outline

Renormalization of bulk/surface functionals

Alternative Renormalization Scheme: Kounterterms

Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Renormalization of Codimension-2 Functionals

Outline

Renormalization of bulk/surface functionals

Alternative Renormalization Scheme: Kounterterms

Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Renormalization of Codimension-2 Functionals

Outline

Renormalization of bulk/surface functionals

Alternative Renormalization Scheme: Kounterterms

Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Renormalization of Codimension-2 Functionals

Outline

- Renormalization of bulk/surface functionals
- Alternative Renormalization Scheme: Kounterterms
- Counterterms and Kounterterms in AdS gravity
- Beyond Kounterterms: Conformal Renormalization
- Renormalization of Codimension-2 Functionals

Outline

Renormalization of bulk/surface functionals

Alternative Renormalization Scheme: Kounterterms

Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Renormalization of Codimension-2 Functionals

Outline

Renormalization of bulk/surface functionals

Alternative Renormalization Scheme: Kounterterms

Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Renormalization of Codimension-2 Functionals

Outline

Renormalization of bulk/surface functionals

Alternative Renormalization Scheme: Kounterterms

Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Renormalization of Codimension-2 Functionals

Outline

Renormalization of bulk/surface functionals

Alternative Renormalization Scheme: Kounterterms

Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Renormalization of Codimension-2 Functionals

Outline

Renormalization of bulk/surface functionals

Alternative Renormalization Scheme: Kounterterms

Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Renormalization of Codimension-2 Functionals

Black Hole Thermodynamics in AdS gravity

Euclidean static black hole metric

$$ds^2 = f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{D-2}^2\,, \qquad f^2(r) = 1 - \frac{2\omega_D GM}{r^{D-3}} + \frac{r^2}{\ell^2}$$

Einstein-AdS gravity

$$I_{EH} = \frac{1}{16\pi G}\int\limits_M d^D x\sqrt{-g}\,(R - 2\Lambda)\,, \qquad \Lambda = -\frac{(D-1)(D-2)}{2\ell^2}$$

Gibbs free energy $G = TI^E$

$$TI_{bulk}^E = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

Black Hole Thermodynamics in AdS gravity

Euclidean static black hole metric

$$ds^2 = f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega^2_{D-2}\,, \qquad f^2(r) = 1 - \frac{2\omega_D GM}{r^{D-3}} + \frac{r^2}{\ell^2}$$

Einstein-AdS gravity

$$I_{EH} = \frac{1}{16\pi G}\int\limits_M d^Dx\sqrt{-g}\,(R-2\Lambda)\,, \qquad \Lambda = -\frac{(D-1)(D-2)}{2\ell^2}$$

Gibbs free energy $G = TI^E$

$$TI^E_{bulk} = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

Black Hole Thermodynamics in AdS gravity

Euclidean static black hole metric

$$ds^2 = f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{D-2}^2 \,, \qquad f^2(r) = 1 - \frac{2\omega_D GM}{r^{D-3}} + \frac{r^2}{\ell^2}$$

Einstein-AdS gravity

$$I_{EH} = \frac{1}{16\pi G}\int_M d^D x\,\sqrt{-g}\,(R - 2\Lambda)\,, \qquad \Lambda = -\frac{(D-1)(D-2)}{2\ell^2}$$

Gibbs free energy $G = TI^E$

$$TI^E_{bulk} = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

Black Hole Thermodynamics in AdS gravity

Euclidean static black hole metric

$$ds^2 = f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{D-2}^2 \,, \qquad f^2(r) = 1 - \frac{2\omega_D GM}{r^{D-3}} + \frac{r^2}{\ell^2}$$

Einstein-AdS gravity

$$I_{EH} = \frac{1}{16\pi G} \int\limits_M d^D x \sqrt{-g}\, (R - 2\Lambda) \,, \qquad \Lambda = -\frac{(D-1)(D-2)}{2\ell^2}$$

Gibbs free energy $G = TI^E$

$$TI^E_{bulk} = \frac{(D-3)}{(D-2)} M - TS + \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Black Hole Thermodynamics in AdS gravity

Euclidean static black hole metric

$$ds^2 = f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{D-2}^2\,, \quad f^2(r) = 1 - \frac{2\omega_D GM}{r^{D-3}} + \frac{r^2}{\ell^2}$$

Einstein-AdS gravity

$$I_{EH} = \frac{1}{16\pi G}\int\limits_M d^D x\sqrt{-g}\,(R-2\Lambda)\,, \qquad \Lambda = -\frac{(D-1)(D-2)}{2\ell^2}$$

Gibbs free energy $G = TI^E$

$$TI_{bulk}^E = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

Black Hole Thermodynamics in AdS gravity

HOLOGRAPHYCL

Euclidean static black hole metric

$$ds^2 = f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{D-2}^2 \,, \qquad f^2(r) = 1 - \frac{2\omega_D GM}{r^{D-3}} + \frac{r^2}{\ell^2}$$

Einstein-AdS gravity

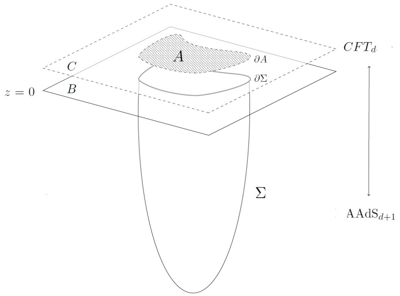
$$I_{EH} = \frac{1}{16\pi G}\int\limits_M d^D x \sqrt{-g}\,(R - 2\Lambda)\,, \qquad \Lambda = -\frac{(D-1)(D-2)}{2\ell^2}$$

Gibbs free energy $G = TI^E$

$$TI_{bulk}^E = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

Energy functionals

Holographic Entanglement Entropy from Minimal Surface [Ryu-Takayanagi, 2006]

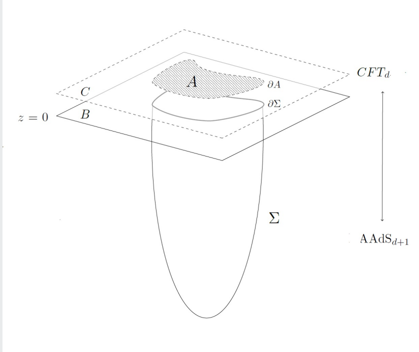


HEE from a Cosmic Brane [Lewkowycz-Maldacena, 2013]

$$S = -\partial_\alpha \lim_{\alpha \to 1} I_{\text{grav}}^{(\alpha)}$$

Energy functionals

Holographic Entanglement Entropy from Minimal Surface [Ryu-Takayanagi, 2006]

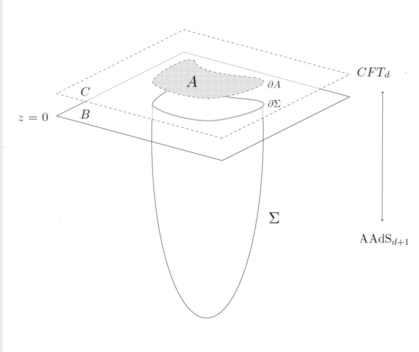


HEE from a Cosmic Brane [Lewkowycz-Maldacena, 2013]

$$S = -\partial_\alpha \lim_{\alpha\to 1} I_{\text{grav}}^{(\alpha)}$$

Energy functionals

Holographic Entanglement Entropy from Minimal Surface [Ryu-Takayanagi, 2006]

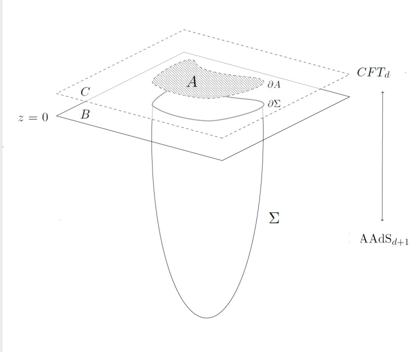


HEE from a Cosmic Brane [Lewkowycz-Maldacena, 2013]

$$S = -\partial_\alpha \lim_{\alpha \to 1} I_{\text{grav}}^{(\alpha)}$$

Energy functionals

Holographic Entanglement Entropy from Minimal Surface [Ryu-Takayanagi, 2006]



HEE from a Cosmic Brane [Lewkowycz-Maldacena, 2013]

$$S = -\partial_\alpha \lim_{\alpha\to 1} I^{(\alpha)}_{\text{grav}}$$

Energy functionals

Bulk gravity action evaluated in conical defects

$$I_{\mathrm{EH}}^{(\alpha)}=\frac{1}{16\pi G}\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}R^{(\alpha)}=\frac{1}{16\pi G}\int\limits_{M} d^4x\sqrt{g}R+\frac{(1-\alpha)}{4G}\mathcal{A}\left[\Sigma\right]$$

Area functional (codimension-2)

$$\mathcal{A}\left[\Sigma\right]=\int\limits_{\Sigma} d^2y\sqrt{\gamma}$$

HEE dual to Einstein gravity

$$S=\frac{1}{4G}\mathcal{A}\left[\Sigma\right]$$

Energy functionals

Bulk gravity action evaluated in conical defects

$$I_{\mathsf{EH}}^{(\alpha)} = \frac{1}{16\pi G} \int\limits_{M^{(\alpha)}} d^4x \sqrt{g} R^{(\alpha)} = \frac{1}{16\pi G} \int\limits_{M} d^4x \sqrt{g} R + \frac{(1-\alpha)}{4G} \mathcal{A}\left[\Sigma\right]$$

Area functional (codimension-2)

$$\mathcal{A}\left[\Sigma\right] = \int\limits_{\Sigma} d^2y \sqrt{\gamma}$$

HEE dual to Einstein gravity

$$S = \frac{1}{4G} \mathcal{A}\left[\Sigma\right]$$

Energy functionals

Bulk gravity action evaluated in conical defects

$$I_{\mathsf{EH}}^{(\alpha)} = \frac{1}{16\pi G} \int\limits_{M^{(\alpha)}} d^4x\sqrt{g}R^{(\alpha)} = \frac{1}{16\pi G} \int\limits_{M} d^4x\sqrt{g}R + \frac{(1-\alpha)}{4G}\mathcal{A}\left[\Sigma\right]$$

Area Functional (codimension-2)

$$\mathcal{A}\left[\Sigma\right] = \int\limits_{\Sigma} d^2y\sqrt{\gamma}$$

HEE dual to Einstein gravity

$$S = \frac{1}{4G}\mathcal{A}\left[\Sigma\right]$$

Energy functionals

Bulk gravity action evaluated in conical defects

$$I_{\mathsf{EH}}^{(\alpha)} = \frac{1}{16\pi G}\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}R^{(\alpha)} = \frac{1}{16\pi G}\int\limits_{M} d^4x\sqrt{g}R + \frac{(1-\alpha)}{4G}\mathcal{A}\left[\Sigma\right]$$

Area Functional (codimension-2)

$$\mathcal{A}\left[\Sigma\right] = \int\limits_{\Sigma} d^2y\sqrt{\gamma}$$

HEE dual to Einstein gravity

$$S = \frac{1}{4G}\mathcal{A}\left[\Sigma\right]$$

Energy functionals

Bulk gravity action evaluated in conical defects

$$I_{\mathsf{EH}}^{(\alpha)} = \frac{1}{16\pi G}\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}R^{(\alpha)} = \frac{1}{16\pi G}\int\limits_{M} d^4x\sqrt{g}R + \frac{(1-\alpha)}{4G}\mathcal{A}\left[\Sigma\right]$$

Area Functional (codimension-2)

$$\mathcal{A}\left[\Sigma\right] = \int\limits_{\Sigma} d^2y\sqrt{\gamma}$$

HEE dual to Einstein gravity

$$S = \frac{1}{4G}\mathcal{A}\left[\Sigma\right]$$

Energy functionals

Bulk gravity action evaluated in conical defects

$$I_{\text{EH}}^{(\alpha)} = \frac{1}{16\pi G} \int\limits_{M^{(\alpha)}} d^4x\sqrt{g}R^{(\alpha)} = \frac{1}{16\pi G} \int\limits_{M} d^4x\sqrt{g}R + \frac{(1-\alpha)}{4G}\mathcal{A}\left[\Sigma\right]$$

Area Functional (codimension-2)

$$\mathcal{A}\left[\Sigma\right] = \int\limits_{\Sigma} d^2y\sqrt{\gamma}$$

HEE dual to Einstein gravity

$$S = \frac{1}{4G}\mathcal{A}\left[\Sigma\right]$$

Counterterms in AdS gravity

Holographic Renormalization [Henningson and Skenderis, 1998]

$$I_{\text{ren}} = I_{EH} - \frac{1}{8\pi G} \int\limits_{\partial M} d^d x \sqrt{-h}\, K + \int\limits_{\partial M} d^d x\, L_{ct}(h, \mathcal{R}, \nabla\mathcal{R})$$

Counterterms [Balasubramanian-Kraus, 1999], [Emparan, Johnson, Myers, 1999]

$$\begin{aligned} 8\pi G\, L_{ct} = {} & \frac{d-1}{\ell}\sqrt{-h} + \frac{\ell\sqrt{-h}}{2(d-2)}\mathcal{R} + \frac{\ell^3\sqrt{-h}}{2(d-2)^2(d-4)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d}{4(d-1)}\mathcal{R}^2\right) \\ & + \frac{\ell^5\sqrt{-h}}{(d-2)^3(d-4)(d-6)}\left(\frac{3d-2}{4(d-1)}\mathcal{R}\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d(d+2)}{16(d-1)^2}\mathcal{R}^3 \right. \\ & \left. -2\mathcal{R}^{ij}\mathcal{R}^{kl}\mathcal{R}_{ijkl} - \frac{d}{4(d-1)}\nabla_i\mathcal{R}\nabla^i\mathcal{R} + \nabla^k\mathcal{R}^{ij}\nabla_k\mathcal{R}_{ij}\right) + \ldots \end{aligned}$$

Counterterms in AdS gravity

Holographic Renormalization [Henningson and Skenderis, 1998]

$$I_{\text{ren}} = I_{EH} - \frac{1}{8\pi G}\int\limits_{\partial M} d^d x\,\sqrt{-h}\,K + \int\limits_{\partial M} d^d x\, L_{ct}(h,\mathcal{R},\nabla\mathcal{R})$$

Counterterms [Balasubramanian-Kraus, 1999], [Emparan, Johnson, Myers, 1999]

$$8\pi G\, L_{ct} = \frac{d-1}{\ell}\sqrt{-h} + \frac{\ell\sqrt{-h}}{2(d-2)}\mathcal{R} + \frac{\ell^3\sqrt{-h}}{2(d-2)^2(d-4)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d}{4(d-1)}\mathcal{R}^2\right)$$
$$+\frac{\ell^5\sqrt{-h}}{(d-2)^3(d-4)(d-6)}\left(\frac{3d-2}{4(d-1)}\mathcal{R}\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d(d+2)}{16(d-1)^2}\mathcal{R}^3\right.$$
$$\left.-2\mathcal{R}^{ij}\mathcal{R}^{kl}\mathcal{R}_{ijkl} - \frac{d}{4(d-1)}\nabla_i\mathcal{R}\nabla^i\mathcal{R} + \nabla^k\mathcal{R}^{ij}\nabla_k\mathcal{R}_{ij}\right) + \ldots$$

Counterterms in AdS gravity

Holographic Renormalization [Henningson and Skenderis, 1998]

$$I_{\text{ren}} = I_{EH} - \frac{1}{8\pi G}\int\limits_{\partial M} d^d x\,\sqrt{-h}\,K + \int\limits_{\partial M} d^d x\, L_{ct}(h,\mathcal{R},\nabla\mathcal{R})$$

Counterterms [Balasubramanian-Kraus, 1999], [Emparan, Johnson, Myers, 1999]

$$\begin{aligned} 8\pi G\, L_{ct} = {} & \tfrac{d-1}{\ell}\sqrt{-h} + \tfrac{\ell\sqrt{-h}}{2(d-2)}\mathcal{R} + \tfrac{\ell^3\sqrt{-h}}{2(d-2)^2(d-4)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \tfrac{d}{4(d-1)}\mathcal{R}^2\right) \\ & + \tfrac{\ell^5\sqrt{-h}}{(d-2)^3(d-4)(d-6)}\left(\tfrac{3d-2}{4(d-1)}\mathcal{R}\mathcal{R}^{ij}\mathcal{R}_{ij} - \tfrac{d(d+2)}{16(d-1)^2}\mathcal{R}^3\right. \\ & \left. -2\mathcal{R}^{ij}\mathcal{R}^{kl}\mathcal{R}_{ijkl} - \tfrac{d}{4(d-1)}\nabla_i\mathcal{R}\nabla^i\mathcal{R} + \nabla^k\mathcal{R}^{ij}\nabla_k\mathcal{R}_{ij}\right) + \ldots \end{aligned}$$

Counterterms in AdS gravity

HOLOGRAPHYCL

Holographic Renormalization [Henningson and Skenderis, 1998]

$$I_{\mathrm{ren}} = I_{EH} - \frac{1}{8\pi G}\int\limits_{\partial M} d^d x\,\sqrt{-h}\,K + \int\limits_{\partial M} d^d x\, L_{ct}(h,\mathcal{R},\nabla\mathcal{R})$$

Counterterms [Balasubramanian-Kraus, 1999], [Emparan, Johnson, Myers, 1999]

$$\begin{aligned}
8\pi G\, L_{ct} = {} & \tfrac{d-1}{\ell}\sqrt{-h} + \tfrac{\ell\sqrt{-h}}{2(d-2)}\mathcal{R} + \tfrac{\ell^3\sqrt{-h}}{2(d-2)^2(d-4)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \tfrac{d}{4(d-1)}\mathcal{R}^2\right)\\
& + \tfrac{\ell^5\sqrt{-h}}{(d-2)^3(d-4)(d-6)}\left(\tfrac{3d-2}{4(d-1)}\mathcal{R}\mathcal{R}^{ij}\mathcal{R}_{ij} - \tfrac{d(d+2)}{16(d-1)^2}\mathcal{R}^3\right.\\
& \left. -2\mathcal{R}^{ij}\mathcal{R}^{kl}\mathcal{R}_{ijkl} - \tfrac{d}{4(d-1)}\nabla_i\mathcal{R}\nabla^i\mathcal{R} + \nabla^k\mathcal{R}^{ij}\nabla_k\mathcal{R}_{ij}\right) + \dots
\end{aligned}$$

Black Hole Thermodynamics and Counterterms

Counterterm Method reproduces BH Thermo

$$G = U - TS$$

Internal Energy

$$U = M + E_0$$

Vacuum Energy in $D = 2n+1$ dimensions

$$E_0 = (-1)^n \frac{(2n-1)!!^2}{(2n)!} \frac{\mathrm{Vol}(S^{2n-1})}{8\pi G} \ell^{2n-2}$$

Black Hole Thermodynamics and Counterterms

Counterterm Method reproduces BH Thermo

$$G = U - TS$$

Internal Energy

$$U = M + E_0$$

Vacuum Energy in $D = 2n + 1$ dimensions

$$E_0 = (-1)^n \frac{(2n-1)!!^2}{(2n)!} \frac{\mathrm{Vol}(S^{2n-1})}{8\pi G} \ell^{2n-2}$$

Black Hole Thermodynamics and Counterterms

Counterterm Method reproduces BH Thermo

$$G = U - TS$$

Internal Energy

$$U = M + E_0$$

Vacuum Energy in $D = 2n + 1$ dimensions

$$E_0 = (-1)^n \frac{(2n-1)!!^2}{(2n)!} \frac{\mathrm{Vol}(S^{2n-1})}{8\pi G} \ell^{2n-2}$$

Black Hole Thermodynamics and Counterterms

Counterterm Method reproduces BH Thermo

$$G = U - TS$$

Internal Energy

$$U = M + E_0$$

Vacuum Energy in $D = 2n + 1$ dimensions

$$E_0 = (-1)^n \frac{(2n-1)!!^2}{(2n)!} \frac{\mathrm{Vol}(S^{2n-1})}{8\pi G} \ell^{2n-2}$$

Black Hole Thermodynamics and Counterterms

Counterterm Method reproduces BH Thermo

$$G = U - TS$$

Internal Energy

$$U = M + E_0$$

Vacuum Energy in $D = 2n + 1$ dimensions

$$E_0 = (-1)^n \frac{(2n-1)!!^2}{(2n)!} \frac{\mathrm{Vol}(S^{2n-1})}{8\pi G} \ell^{2n-2}$$

Black Hole Thermodynamics and Counterterms

Counterterm Method reproduces BH Thermo

$$G = U - TS$$

Internal Energy

$$U = M + E_0$$

Vacuum Energy in $D = 2n + 1$ dimensions

$$E_0 = (-1)^n \frac{(2n-1)!!^2}{(2n)!} \frac{\mathrm{Vol}(S^{2n-1})}{8\pi G} \ell^{2n-2}$$

Kounterterms in AdS gravity

Extrinsic counterterms

$$\tilde{I}_{ren} = I + c_d \int\limits_{\partial M} d^d x \, B_d(h, K, \mathcal{R})$$

Kounterterms = counterterms of unusual sort (depend on K_{ij} and $\mathcal{R}^{kl}_{ij}(h)$)

$D = 2n$ dimensions [R.O., hep-th/0504233]

$$B_{2n-1} = 2n\sqrt{-h} \int\limits_0^1 dt \; \delta^{[i_1 \cdots i_{2n-1}]}_{[j_1 \cdots j_{2n-1}]} K^{j_1}_{i_1} \left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3} - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3} \right) \times \cdots$$

$$\cdots \times \left(\frac{1}{2}\mathcal{R}^{j_{2n-2} j_{2n-1}}_{i_{2n-2} i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}} K^{j_{2n-1}}_{i_{2n-1}} \right)$$

Kounterterms in AdS gravity

Extrinsic counterterms

$$\tilde{I}_{ren} = I + c_d \int\limits_{\partial M} d^d x \, B_d(h, K, \mathcal{R})$$

Kounterterms = counterterms of unusual sort (depend on K_{ij} and $\mathcal{R}^{kl}_{ij}(h)$)

$D = 2n$ dimensions [R. O. , hep-th/0504233]

$$B_{2n-1} = 2n\sqrt{-h} \int\limits_0^1 dt \; \delta^{[i_1 \cdots i_{2n-1}]}_{[j_1 \cdots j_{2n-1}]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3} - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3} \right) \times \cdots$$

$$\cdots \times \left(\frac{1}{2} \mathcal{R}^{j_{2n-2} j_{2n-1}}_{i_{2n-2} i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}} K^{j_{2n-1}}_{i_{2n-1}} \right)$$

Kounterterms in AdS gravity

Extrinsic counterterms

$$\tilde{I}_{ren} = I + c_d \int\limits_{\partial M} d^d x \, B_d(h, K, \mathcal{R})$$

Kounterterms = counterterms of unusual sort (depend on K_{ij} and $\mathcal{R}^{kl}_{ij}(h)$)

Kounterterms in AdS gravity

Extrinsic counterterms

$$\tilde{I}_{ren} = I + c_d \int\limits_{\partial M} d^d x \, B_d(h, K, \mathcal{R})$$

Kounterterms $=$ counterterms of unusual sort (depend on K_{ij} and $\mathcal{R}^{kl}_{ij}(h)$)

$D = 2n$ dimensions [R.O., hep-th/0504233]

$$B_{2n-1} = 2n\sqrt{-h}\int\limits_0^1 dt\ \delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3} - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3}\right) \times \cdots$$

$$\cdots \times \left(\frac{1}{2}\mathcal{R}^{j_{2n-2} j_{2n-1}}_{i_{2n-2} i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}} K^{j_{2n-1}}_{i_{2n-1}}\right)$$

Kounterterms in AdS gravity

Extrinsic counterterms

$$\tilde{I}_{ren} = I + c_d \int\limits_{\partial M} d^d x \, B_d(h, K, \mathcal{R})$$

Kounterterms $=$ counterterms of unusual sort (depend on K_{ij} and $\mathcal{R}^{kl}_{ij}(h)$)

$D = 2n$ dimensions [R.O., hep-th/0504233]

$$B_{2n-1} = 2n\sqrt{-h} \int\limits_0^1 dt \; \delta^{[i_1 \cdots i_{2n-1}]}_{[j_1 \cdots j_{2n-1}]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3} - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3} \right) \times \cdots$$

$$\cdots \times \left(\frac{1}{2} \mathcal{R}^{j_{2n-2} j_{2n-1}}_{i_{2n-2} i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}} K^{j_{2n-1}}_{i_{2n-1}} \right)$$

Kounterterms in AdS gravity

Kounterterms in $D = 2n+1$ [R.O., hep-th/0610230]

$$
\begin{aligned}
B_{2n} \;=\; & 2n\sqrt{-h}\int\limits_0^1 dt\int\limits_0^t ds\,\delta^{[j_1\cdots j_{2n}]}_{[i_1\cdots i_{2n}]}\,K^{i_1}_{j_1}\delta^{i_2}_{j_2}\left(\frac{1}{2}\,\mathcal{R}^{i_3i_4}_{j_3j_4}-t^2K^{i_3}_{j_3}K^{i_4}_{j_4}+\frac{s^2}{\ell^2}\,\delta^{i_3}_{j_3}\delta^{i_4}_{j_4}\right)\times\cdots \\
& \cdots\times\left(\frac{1}{2}\,\mathcal{R}^{i_{2n-1}i_{2n}}_{j_{2n-1}j_{2n}}-t^2K^{i_{2n-1}}_{j_{2n-1}}K^{i_{2n}}_{j_{2n}}+\frac{s^2}{\ell^2}\,\delta^{i_{2n-1}}_{j_{2n-1}}\delta^{i_{2n}}_{j_{2n}}\right).
\end{aligned}
$$

Kounterterms in AdS gravity

In $D=2n$ dimensions

$$\begin{aligned} \mathrm{tr}(F^n) &= dL^{CS}_{2n-1}(A) \\ F &= dA + A\wedge A \end{aligned}$$

Explicit realization of Chern-Simons forms

$$L^{CS}_{2n-1}(A) = n\int_0^1 dt\,\mathrm{tr}\left[AF_t^{n-1}\right] \qquad F_t = t\,dA + t^2A^2$$

Global issues (topology)

$$\int_{M_{2n}} (\mathrm{Euler})_{2n} = (4\pi)^n n!\,\chi(M_{2n}) + \int_{\partial M_{2n}} B_{2n-1}$$

Kounterterms in AdS gravity

In $D = 2n$ dimensions

$$\begin{aligned} \mathrm{tr}(F^n) &= dL^{CS}_{2n-1}(A) \\ F &= dA + A\wedge A \end{aligned}$$

Explicit realization of Chern-Simons forms

$$L^{CS}_{2n-1}(A) = n\int_0^1 dt\, \mathrm{tr}\left[AF_t^{n-1}\right] \qquad F_t = t\,dA + t^2A^2$$

Global issues (topology)

$$\int_{M_{2n}} (\mathrm{Euler})_{2n} = (4\pi)^n n!\,\chi(M_{2n}) + \int_{\partial M_{2n}} B_{2n-1}$$

Kounterterms in AdS gravity

In $D = 2n$ dimensions

$$\begin{aligned} \mathrm{tr}(F^n) &= dL^{CS}_{2n-1}(A) \\ F &= dA + A \wedge A \end{aligned}$$

Explicit realization of Chern-Simons forms

$$L^{CS}_{2n-1}(A) = n \int_0^1 dt \, \mathrm{tr}\left[A F_t^{n-1} \right] \qquad F_t = t dA + t^2 A^2$$

Global issues: topology?

$$\int_{M_{2n}} (\mathrm{Euler})_{2n} = (4\pi)^n n! \chi(M_{2n}) + \int_{\partial M_{2n}} B_{2n-1}$$

Kounterterms in AdS gravity

In $D = 2n$ dimensions

$$\begin{aligned} \mathrm{tr}(F^n) &= dL^{CS}_{2n-1}(A) \\ F &= dA + A \wedge A \end{aligned}$$

Explicit realization of Chern-Simons forms

$$L^{CS}_{2n-1}(A) = n \int_0^1 dt\, \mathrm{tr}\left[A F_t^{n-1} \right] \qquad F_t = t dA + t^2 A^2$$

Global issues (topology)

$$\int_{M_{2n}} (\mathrm{Euler})_{2n} = (4\pi)^n n! \chi(M_{2n}) + \int_{\partial M_{2n}} B_{2n-1}$$

Kounterterms in AdS gravity

In $D = 2n$ dimensions

$$\begin{aligned} \mathrm{tr}(F^n) &= dL_{2n-1}^{CS}(A) \\ F &= dA + A \wedge A \end{aligned}$$

Explicit realization of Chern-Simons forms

$$L_{2n-1}^{CS}(A) = n \int_0^1 dt \, \mathrm{tr}\left[A F_t^{n-1} \right] \qquad F_t = t dA + t^2 A^2$$

Global issues (topology)

$$\int_{M_{2n}} (\mathrm{Euler})_{2n} = (4\pi)^n n! \chi(M_{2n}) + \int_{\partial M_{2n}} B_{2n-1}$$

Kounterterms in AdS gravity

$D = 2n+1$ dimensions

$$\begin{aligned} L^{TF}_{2n+1}(A,\bar{A}) &= (n+1)\int_0^1 dt\, \mathrm{tr}\left[\left(A-\bar{A}\right)F_t^n\right] \\ F_t &= dA_t + A_t^2\,, \quad A_t = tA + (1-t)\bar{A} \end{aligned}$$

Gauge-invariant extension of CS forms $L^{TF}_{2n+1}(A,\bar{A}) = L^{CS}_{2n+1}(A) - L^{CS}_{2n+1}(\bar{A}) + d\beta_{2n}(A,\bar{A})$

Contact term

$$\begin{aligned} \beta_{2n}(A,\bar{A}) &= \int_0^1 dt \int_0^t ds\, \mathrm{tr}\left[A_t\left(A-\bar{A}\right)F_{st}^{n-1}\right] \\ F_{st} &= sF_t + s(s-1)A_t^2 \end{aligned}$$

Kounterterms in AdS gravity

$D = 2n+1$ dimensions

$$
\begin{aligned}
L_{2n+1}^{TF}(A,\bar{A}) &= (n+1)\int_0^1 dt\, \mathrm{tr}\left[\left(A-\bar{A}\right)F_t^n\right] \\
F_t &= dA_t + A_t^2\,, \quad A_t = tA + (1-t)\bar{A}
\end{aligned}
$$

Gauge-invariant extension of CS forms $L_{2n+1}^{TF}(A,\bar{A}) = L_{2n+1}^{CS}(A) - L_{2n+1}^{CS}(\bar{A}) + d\beta_{2n}(A,\bar{A})$

Contact term

$$
\begin{aligned}
\beta_{2n}(A,\bar{A}) &= \int_0^1 dt \int_0^t ds\, \mathrm{tr}\left[A_t\left(A-\bar{A}\right)F_{st}^{n-1}\right] \\
F_{st} &= sF_t + s(s-1)A_t^2
\end{aligned}
$$

Kounterterms in AdS gravity

$D = 2n+1$ dimensions

$$
\begin{aligned}
L^{TF}_{2n+1}(A,\bar{A}) &= (n+1)\int_0^1 dt\, \mathrm{tr}\left[\left(A-\bar{A}\right)F_t^n\right] \\
F_t &= dA_t + A_t^2\,, \qquad A_t = tA + (1-t)\bar{A}
\end{aligned}
$$

Gauge-invariant extension of CS forms $L^{TF}_{2n+1}(A,\bar{A}) = L^{CS}_{2n+1}(A) - L^{CS}_{2n+1}(\bar{A}) + d\beta_{2n}(A,\bar{A})$

Contact term

$$
\begin{aligned}
\beta_{2n}(A,\bar{A}) &= \int_0^1 dt \int_0^t ds\, \mathrm{tr}\left[A_t\left(A-\bar{A}\right)F_{st}^{n-1}\right] \\
F_{st} &= sF_t + s(s-1)A_t^2
\end{aligned}
$$

Kounterterms in AdS gravity

$D = 2n+1$ dimensions

$$
\begin{aligned}
L_{2n+1}^{TF}(A,\bar{A}) &= (n+1)\int_0^1 dt\, \mathrm{tr}\left[\left(A-\bar{A}\right)F_t^n\right] \\
F_t &= dA_t + A_t^2\,, \quad A_t = tA + (1-t)\bar{A}
\end{aligned}
$$

Gauge-invariant extension of CS forms $L_{2n+1}^{TF}(A,\bar{A}) = L_{2n+1}^{CS}(A) - L_{2n+1}^{CS}(\bar{A}) + d\beta_{2n}(A,\bar{A})$

Contact term

$$
\begin{aligned}
\beta_{2n}(A,\bar{A}) &= \int_0^1 dt \int_0^t ds\, \mathrm{tr}\left[A_t\left(A-\bar{A}\right)F_{st}^{n-1}\right] \\
F_{st} &= sF_t + s(s-1)A_t^2
\end{aligned}
$$

Kounterterms in AdS gravity

$D = 2n+1$ dimensions

$$L^{TF}_{2n+1}(A,\bar{A}) = (n+1)\int_0^1 dt\,\mathrm{tr}\left[\left(A-\bar{A}\right)F_t^n\right]$$

$$F_t = dA_t + A_t^2\,, \quad A_t = tA + (1-t)\bar{A}$$

Gauge-invariant extension of CS forms $L^{TF}_{2n+1}(A,\bar{A}) = L^{CS}_{2n+1}(A) - L^{CS}_{2n+1}(\bar{A}) + d\beta_{2n}(A,\bar{A})$

Contact term

$$\beta_{2n}(A,\bar{A}) = \int_0^1 dt\int_0^t ds\,\mathrm{tr}\left[A_t\left(A-\bar{A}\right)F_{st}^{n-1}\right]$$

$$F_{st} = sF_t + s(s-1)A_t^2$$

Renormalized Action = Renormalized Volume

Black Hole Thermodynamics

$$TI^E_{bulk} = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

Euclidean Kounterterms

$$T\, c_d \int\limits_{\partial M} B_d = \frac{M}{(D-2)} + E_0 - \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

Correct Black Hole Thermo with $U = M + E_0$

Renormalized Action = Renormalized Volume

Black Hole Thermodynamics

$$TI_{bulk}^{E} = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

Euclidean Kounterterms

$$T\, c_d \int\limits_{\partial M} B_d = \frac{M}{(D-2)} + E_0 - \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

Correct Black Hole Thermo with $U = M + E_0$

Renormalized Action = Renormalized Volume

Black Hole Thermodynamics

$$T I^E_{bulk} = \frac{(D-3)}{(D-2)} M - TS + \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Euclidean Kounterterms

$$T\, c_d \int_{\partial M} B_d = \frac{M}{(D-2)} + E_0 - \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Correct Black Hole Thermo with $U = M + E_0$

Renormalized Action = Renormalized Volume

HOLOGRAPHYCL

Black Hole Thermodynamics

$$TI_{bulk}^{E} = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Euclidean Kounterterms

$$T\, c_d \int\limits_{\partial M} B_d = \frac{M}{(D-2)} + E_0 - \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Correct Black Hole Thermo with $U = M + E_0$

Renormalized Action = Renormalized Volume

HOLOGRAPHYCL

Black Hole Thermodynamics

$$T I_{bulk}^{E} = \frac{(D-3)}{(D-2)} M - TS + \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Euclidean Kounterterms

$$T\, c_d \int_{\partial M} B_d = \frac{M}{(D-2)} + E_0 - \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Correct Black Hole Thermo with $U = M + E_0$

Boundary conditions in AdS gravity

HOLOGRAPHYCL

Fefferman-Graham expansion for AAdS Einstein spaces

$$ds^2 = \frac{\ell^2}{z^2}\,dz^2 + \frac{1}{z^2}\,g_{ij}(x,z)\,dx^i dx^j\,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots$$

Dirichlet b.c. $\delta h_{ij} = 0$ does not make sense in AAdS spaces [Papadimitriou and Skenderis, 2004]

$$h_{ij} = \frac{g_{(0)ij}}{z^2} + \ldots$$

Renormalization ↔ variational problem in $g_{(0)ij}$

$$\delta I_{ren} = \frac{1}{2}\int\limits_{\partial M} \sqrt{-g_{(0)}}\,T^{ij}[g_{(0)}]\,\delta g_{(0)ij}$$

Boundary conditions in AdS gravity

Fefferman-Graham expansion for AAdS Einstein spaces

$$ds^2 = \frac{\ell^2}{z^2}\,dz^2 + \frac{1}{z^2}\,g_{ij}(x,z)\,dx^i dx^j\,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots$$

Dirichlet b.c. on h_{ij} — it does not make sense in AAdS spaces [Papadimitriou and Skenderis, 2004]

$$h_{ij} = \frac{g_{(0)ij}}{z^2} + \cdots$$

Renormalization = variational problem in $g_{(0)ij}$

$$\delta I_{ren} = \frac{1}{2}\int\limits_{\partial M} \sqrt{-g_{(0)}}\,T^{ij}[g_{(0)}]\,\delta g_{(0)ij}$$

Boundary conditions in AdS gravity

Fefferman-Graham expansion for AAdS Einstein spaces

$$ds^2 = \frac{\ell^2}{z^2}\,dz^2 + \frac{1}{z^2}\,g_{ij}(x,z)\,dx^i dx^j\,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots$$

Dirichlet b.c. $\delta h_{ij} = 0$ does not make sense in AAdS spaces [Papadimitriou and Skenderis, 2004]

$$h_{ij} = \frac{g_{(0)ij}}{z^2} + \ldots$$

Renormalization = variational problem in $g_{(0)ij}$

$$\delta I_{ren} = \frac{1}{2}\int\limits_{\partial M} \sqrt{-g_{(0)}}\,T^{ij}[g_{(0)}]\,\delta g_{(0)ij}$$

Boundary conditions in AdS gravity

Fefferman-Graham expansion for AAdS Einstein spaces

$$ds^2 = \frac{\ell^2}{z^2}\,dz^2 + \frac{1}{z^2}\,g_{ij}(x,z)\,dx^i dx^j\,, \qquad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots$$

Dirichlet b.c. $\delta h_{ij} = 0$ does not make sense in AAdS spaces [Papadimitriou and Skenderis, 2004]

$$h_{ij} = \frac{g_{(0)ij}}{z^2} + \ldots$$

Renormalization = variational problem in $g_{(0)ij}$

$$\delta I_{ren} = \frac{1}{2}\int\limits_{\partial M} \sqrt{-g_{(0)}}\,T^{ij}[g_{(0)}]\,\delta g_{(0)ij}$$

Boundary conditions in AdS gravity

Fefferman-Graham expansion for AAdS Einstein spaces

$$ds^2 = \frac{\ell^2}{z^2}\,dz^2 + \frac{1}{z^2}\,g_{ij}(x,z)\,dx^i dx^j\,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots$$

Dirichlet b.c. $\delta h_{ij} = 0$ does not make sense in AAdS spaces [Papadimitriou and Skenderis, 2004]

$$h_{ij} = \frac{g_{(0)ij}}{z^2} + \ldots$$

Renormalization = variational problem in $g_{(0)ij}$

$$\delta I_{ren} = \frac{1}{2}\int\limits_{\partial M} \sqrt{-g_{(0)}}\,T^{ij}[g_{(0)}]\,\delta g_{(0)ij}$$

Kounterterms and Holography

Asymptotic form of the extrinsic curvature

$$K_{ij} = \frac{1}{\ell}\frac{g_{(0)ij}}{z^2} + \ldots$$

Counterterms of a different sort ...

$$\tilde{I}_{ren} = I_{EH} + c_d \int\limits_{\partial M} d^d x \, B(f(h), K)$$

... as long as the theory is *holographic*

$$\delta \tilde{I}_{ren} = \frac{1}{2} \int\limits_{\partial M} \sqrt{-g_{(0)}}\, \tau^{ij} \, \delta g_{(0)ij}$$

Kounterterms and Holography

Asymptotic form of the extrinsic curvature

$$K_{ij} = \frac{1}{\ell}\frac{g_{(0)ij}}{z^2} + ...$$

Counterterms of a different sort

$$\tilde{I}_{ren} = I_{EH} + c_d \int\limits_{\partial M} d^d x \, B(f(h), K)$$

... as long as the theory is holographic

$$\delta \tilde{I}_{ren} = \frac{1}{2} \int\limits_{\partial M} \sqrt{-g_{(0)}}\, \tau^{ij} \, \delta g_{(0)ij}$$

Kounterterms and Holography

Asymptotic form of the extrinsic curvature

$$K_{ij} = \frac{1}{\ell}\frac{g_{(0)ij}}{z^2} + \dots$$

Counterterms of a different sort...

$$\tilde{I}_{ren} = I_{EH} + c_d \int\limits_{\partial M} d^d x \, B(f(h), K)$$

...as long as the theory is *holographic*

$$\delta \tilde{I}_{ren} = \frac{1}{2} \int\limits_{\partial M} \sqrt{-g_{(0)}} \tau^{ij} \, \delta g_{(0)ij}$$

Kounterterms and Holography

Asymptotic form of the extrinsic curvature

$$K_{ij} = \frac{1}{\ell}\frac{g_{(0)ij}}{z^2} + \ldots$$

Counterterms of a different sort...

$$\tilde{I}_{ren} = I_{EH} + c_d \int\limits_{\partial M} d^d x \, B(f(h), K)$$

...as long as the theory is *holographic*

$$\delta \tilde{I}_{ren} = \frac{1}{2}\int\limits_{\partial M} \sqrt{-g_{(0)}}\tau^{ij}\,\delta g_{(0)ij}$$

Kounterterms and Holography

Asymptotic form of the extrinsic curvature

$$K_{ij} = \frac{1}{\ell}\frac{g_{(0)ij}}{z^2} + \ldots$$

Counterterms of a different sort...

$$\tilde{I}_{ren} = I_{EH} + c_d \int\limits_{\partial M} d^d x\, B(f(h), K)$$

...as long as the theory is *holographic*

$$\delta\tilde{I}_{ren} = \frac{1}{2}\int\limits_{\partial M} \sqrt{-g_{(0)}}\tau^{ij}\,\delta g_{(0)ij}$$

From extrinsic to intrinsic renormalization in 4D

AdS gravity action + KTs

$$\tilde{I}_{\text{ren}} = I_{EH} + \frac{\ell^2}{16\pi G}\int\limits_{\partial M} d^3x\,\sqrt{-h}\;\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1}\left(\frac{1}{2}\,\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\,K^{j_2}_{i_2}K^{j_3}_{i_3}\right).$$

Adding zero...

$$\tilde{I}_{\text{ren}} = I_{EH} - \frac{1}{8\pi G}\int\limits_{\partial M} d^3x\sqrt{-h}\,K + \int\limits_{\partial M} d^3x\,L_{ct}\,.$$

$$L_{ct} = \frac{\ell^2}{16\pi G}\sqrt{-h}\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1}\left(\frac{1}{2}\,\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\,K^{j_2}_{i_2}K^{j_3}_{i_3} + \frac{1}{\ell^2}\,\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right).$$

Fefferman-Graham expansion

$$K^i_j = \frac{1}{\ell}\delta^i_j - \ell S^i_j(h) + \mathcal{O}(\mathcal{R}^2)\,, \qquad S^i_j(h) = \frac{1}{d-2}\left(\mathcal{R}^i_j(h) - \frac{1}{2(d-1)}\delta^i_j\mathcal{R}(h)\right)$$

From extrinsic to intrinsic renormalization in 4D

AdS gravity action + KTs

$$\tilde{I}_{\text{ren}} = I_{EH} + \frac{\ell^2}{16\pi G}\int\limits_{\partial M} d^3x\,\sqrt{-h}\;\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1}\left(\frac{1}{2}\,\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\,K^{j_2}_{i_2}K^{j_3}_{i_3}\right).$$

Adding zero...

$$\tilde{I}_{\text{ren}} = I_{EH} - \frac{1}{8\pi G}\int\limits_{\partial M} d^3x\sqrt{-h}\,K + \int\limits_{\partial M} d^3x\,L_{ct}\,.$$

$$L_{ct} = \frac{\ell^2}{16\pi G}\sqrt{-h}\,\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1}\left(\frac{1}{2}\,\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\,K^{j_2}_{i_2}K^{j_3}_{i_3} + \frac{1}{\ell^2}\,\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right).$$

Fefferman-Graham expansion

$$K^i_j = \frac{1}{\ell}\delta^i_j - \ell S^i_j(h) + \mathcal{O}(\mathcal{R}^2)\,, \qquad S^i_j(h) = \frac{1}{d-2}\left(\mathcal{R}^i_j(h) - \frac{1}{2(d-1)}\delta^i_j\mathcal{R}(h)\right)$$

From extrinsic to intrinsic renormalization in 4D

AdS gravity action + KTs

$$\tilde{I}_{\mathrm{ren}} = I_{EH} + \frac{\ell^2}{16\pi G} \int\limits_{\partial M} d^3x \sqrt{-h}\, \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} \right).$$

Adding zero...

$$\tilde{I}_{\mathrm{ren}} = I_{EH} - \frac{1}{8\pi G} \int\limits_{\partial M} d^3x \sqrt{-h}\, K + \int\limits_{\partial M} d^3x\, L_{ct}.$$

$$L_{ct} = \frac{\ell^2}{16\pi G} \sqrt{-h} \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} + \frac{1}{\ell^2} \delta^{j_2}_{i_2} \delta^{j_3}_{i_3} \right).$$

Fefferman-Graham expansion

$$K^i_j = \frac{1}{\ell}\delta^i_j - \ell S^i_j(h) + \mathcal{O}(\mathcal{R}^2), \qquad S^i_j(h) = \frac{1}{d-2}\left(\mathcal{R}^i_j(h) - \frac{1}{2(d-1)}\delta^i_j \mathcal{R}(h)\right)$$

From extrinsic to intrinsic renormalization in 4D

AdS gravity action + KTs

$$\tilde{I}_{\text{ren}} = I_{EH} + \frac{\ell^2}{16\pi G}\int\limits_{\partial M} d^3x\,\sqrt{-h}\;\delta^{[i_1i_2i_3]}_{[j_1j_2j_3]}K^{j_1}_{i_1}\left(\frac{1}{2}\,\mathcal{R}^{j_2j_3}_{i_2i_3}(h)-\frac{1}{3}\,K^{j_2}_{i_2}K^{j_3}_{i_3}\right).$$

Adding zero...

$$\tilde{I}_{\text{ren}} = I_{EH} - \frac{1}{8\pi G}\int\limits_{\partial M} d^3x\sqrt{-h}\,K + \int\limits_{\partial M} d^3x\,L_{ct}\,.$$

$$L_{ct} = \frac{\ell^2}{16\pi G}\sqrt{-h}\delta^{[i_1i_2i_3]}_{[j_1j_2j_3]}K^{j_1}_{i_1}\left(\frac{1}{2}\,\mathcal{R}^{j_2j_3}_{i_2i_3}(h)-\frac{1}{3}\,K^{j_2}_{i_2}K^{j_3}_{i_3}+\frac{1}{\ell^2}\,\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right).$$

Fefferman-Graham expansion

$$K^i_j = \frac{1}{\ell}\delta^i_j - \ell S^i_j(h) + \mathcal{O}(\mathcal{R}^2)\,,\qquad S^i_j(h) = \frac{1}{d-2}\left(\mathcal{R}^i_j(h) - \frac{1}{2(d-1)}\delta^i_j\mathcal{R}(h)\right)$$

From extrinsic to intrinsic renormalization in 4D

AdS gravity action + KTs

$$\tilde{I}_{\text{ren}} = I_{EH} + \frac{\ell^2}{16\pi G} \int\limits_{\partial M} d^3x \sqrt{-h}\, \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} \right).$$

Adding zero...

$$\tilde{I}_{\text{ren}} = I_{EH} - \frac{1}{8\pi G} \int\limits_{\partial M} d^3x \sqrt{-h}\, K + \int\limits_{\partial M} d^3x\, L_{ct}.$$

$$L_{ct} = \frac{\ell^2}{16\pi G} \sqrt{-h} \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} + \frac{1}{\ell^2} \delta^{j_2}_{i_2} \delta^{j_3}_{i_3} \right).$$

Fefferman-Graham expansion

$$K^i_j = \frac{1}{\ell} \delta^i_j - \ell S^i_j(h) + \mathcal{O}(\mathcal{R}^2), \qquad S^i_j(h) = \frac{1}{d-2} \left(\mathcal{R}^i_j(h) - \frac{1}{2(d-1)} \delta^i_j \mathcal{R}(h) \right)$$

From extrinsic to intrinsic renormalization

$$L_{ct} = \frac{\ell^2}{16\pi G}\frac{\sqrt{-g}}{z^3}\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]}\left(\frac{\delta^{j_1}_{i_1}}{\ell} - \ell S^{i_1}_{j_1}\right)\times$$
$$\times\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\left(\frac{\delta^{j_2}_{i_2}}{\ell} - \ell S^{i_2}_{j_2}\right)\left(\frac{\delta^{j_3}_{i_3}}{\ell} - \ell S^{i_3}_{j_3}\right) + \frac{1}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) + \ldots$$

Kounterterms turn into counterterms [O.Miskovic and R.O., 0902.2082]

$$L_{ct} = \frac{1}{8\pi G}\frac{\sqrt{-g}}{z^3}\left(\frac{2}{\ell} + \frac{\ell}{2}z^2\mathcal{R}(g)\right) + \mathcal{O}(z)$$
$$= \frac{1}{8\pi G}\sqrt{-h}\left(\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right).$$

From extrinsic to intrinsic renormalization

$$L_{ct} = \frac{\ell^2}{16\pi G}\frac{\sqrt{-g}}{z^3}\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]}\left(\frac{\delta^{j_1}_{i_1}}{\ell} - \ell S^{i_1}_{j_1}\right)\times$$
$$\times\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\left(\frac{\delta^{j_2}_{i_2}}{\ell} - \ell S^{i_2}_{j_2}\right)\left(\frac{\delta^{j_3}_{i_3}}{\ell} - \ell S^{i_3}_{j_3}\right) + \frac{1}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) + \ldots$$

Kounterterms turn into counterterms [O.Miskovic and R.O., 0902.2082]

$$L_{ct} = \frac{1}{8\pi G}\frac{\sqrt{-g}}{z^3}\left(\frac{2}{\ell} + \frac{\ell}{2}z^2\mathcal{R}(g)\right) + \mathcal{O}(z)$$
$$= \frac{1}{8\pi G}\sqrt{-h}\left(\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right).$$

From extrinsic to intrinsic renormalization

$$L_{ct} = \frac{\ell^2}{16\pi G}\frac{\sqrt{-g}}{z^3}\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]}\left(\frac{\delta^{j_1}_{i_1}}{\ell} - \ell S^{i_1}_{j_1}\right)\times$$
$$\times\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\left(\frac{\delta^{j_2}_{i_2}}{\ell} - \ell S^{i_2}_{j_2}\right)\left(\frac{\delta^{j_3}_{i_3}}{\ell} - \ell S^{i_3}_{j_3}\right) + \frac{1}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) + \ldots$$

Kounterterms turn into counterterms [O.Miskovic and R.O., 0902.2082]

$$L_{ct} = \frac{1}{8\pi G}\frac{\sqrt{-g}}{z^3}\left(\frac{2}{\ell} + \frac{\ell}{2}z^2\mathcal{R}(g)\right) + \mathcal{O}(z)$$
$$= \frac{1}{8\pi G}\sqrt{-h}\left(\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right).$$

From extrinsic to intrinsic renormalization

$$
\begin{aligned}
L_{ct} = \frac{\ell^2}{16\pi G}\frac{\sqrt{-g}}{z^3}\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]}\left(\frac{\delta^{j_1}_{i_1}}{\ell} - \ell S^{i_1}_{j_1}\right) \times \\
\times \left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\left(\frac{\delta^{j_2}_{i_2}}{\ell} - \ell S^{i_2}_{j_2}\right)\left(\frac{\delta^{j_3}_{i_3}}{\ell} - \ell S^{i_3}_{j_3}\right) + \frac{1}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) + \ldots
\end{aligned}
$$

Kounterterms turn into counterterms [O.Miskovic and R.O., 0902.2082]

$$
\begin{aligned}
L_{ct} &= \frac{1}{8\pi G}\frac{\sqrt{-g}}{z^3}\left(\frac{2}{\ell} + \frac{\ell}{2}z^2\mathcal{R}(g)\right) + \mathcal{O}(z) \\
&= \frac{1}{8\pi G}\sqrt{-h}\left(\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right) .
\end{aligned}
$$

Higher even dimensions

EH-AdS gravity +KTs

$$\mathrm{B}_{2n-1} = 2n\sqrt{-h}\int_0^1 dt\; \delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]} K^{j_1}_{i_1}\left(\tfrac{1}{2}\mathcal{R}^{j_2j_3}_{i_2i_3} - t^2 K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots$$
$$\cdots\times\left(\tfrac{1}{2}\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}}K^{j_{2n-1}}_{i_{2n-1}}\right).$$

$$\tilde{I}_{\text{ren}} = I_{Dir} + \int_{\partial M} d^{2n-1}x\, L_{ct}$$

$$L_{ct} = c_{2n-1}B_{2n-1} + \frac{1}{8\pi G}\sqrt{-h}K$$
$$= \frac{(-\ell^2)^n}{8\pi G(2n-2)!}\sqrt{-h}\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}K^{j_1}_{i_1}\int_0^1 dt\;\Bigg[\left(\tfrac{1}{2}\mathcal{R}^{j_2j_3}_{i_2i_3} - t^2K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots$$
$$\cdots\times\left(\tfrac{1}{2}\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - t^2K^{j_{2n-2}}_{i_{2n-2}}K^{j_{2n-1}}_{i_{2n-1}}\right) + \frac{(-1)^n}{\ell^{2n-2}}\delta^{j_2}_{i_2}\cdots\delta^{j_{2n-1}}_{i_{2n-1}}\Bigg].$$

EH-AdS gravity +KTs

$$\mathrm{B}_{2n-1} = 2n\sqrt{-h}\int_0^1 dt\ \delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2j_3}_{i_2i_3} - t^2 K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots$$
$$\cdots\times\left(\frac{1}{2}\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}}K^{j_{2n-1}}_{i_{2n-1}}\right).$$

$$\tilde{I}_{\mathrm{ren}} = I_{Dir} + \int_{\partial M} d^{2n-1}x\, L_{ct}$$

Higher even dimensions

EH-AdS gravity +KTs

$$\mathrm{B}_{2n-1} = 2n\sqrt{-h}\int_0^1 dt\ \delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2j_3}_{i_2i_3} - t^2 K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots$$
$$\cdots\times\left(\frac{1}{2}\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}}K^{j_{2n-1}}_{i_{2n-1}}\right).$$

$$\tilde{I}_{\mathrm{ren}} = I_{Dir} + \int_{\partial M} d^{2n-1}x\, L_{ct}$$

$$L_{ct} = c_{2n-1}B_{2n-1} + \frac{1}{8\pi G}\sqrt{-h}K$$
$$= \frac{(-\ell^2)^n}{8\pi G(2n-2)!}\sqrt{-h}\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}K^{j_1}_{i_1}\int_0^1 dt\ \left[\left(\frac{1}{2}\mathcal{R}^{j_2j_3}_{i_2i_3} - t^2K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots\right.$$
$$\left.\cdots\times\left(\frac{1}{2}\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - t^2K^{j_{2n-2}}_{i_{2n-2}}K^{j_{2n-1}}_{i_{2n-1}}\right) + \frac{(-1)^n}{\ell^{2n-2}}\delta^{j_2}_{i_2}\cdots\delta^{j_{2n-1}}_{i_{2n-1}}\right].$$

Higher even dimensions

EH-AdS gravity +KTs

$$\mathsf{B}_{2n-1} = 2n\sqrt{-h}\int\limits_0^1 dt\ \delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2j_3}_{i_2i_3} - t^2 K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots$$
$$\cdots\times\left(\frac{1}{2}\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}}K^{j_{2n-1}}_{i_{2n-1}}\right).$$

$$\tilde{I}_{\mathsf{ren}} = I_{Dir} + \int\limits_{\partial M} d^{2n-1}x\, L_{ct}$$

$$L_{ct} = c_{2n-1}B_{2n-1} + \frac{1}{8\pi G}\sqrt{-h}K$$
$$= \frac{(-\ell^2)^n}{8\pi G(2n-2)!}\sqrt{-h}\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]} K^{j_1}_{i_1}\int\limits_0^1 dt\ \left[\left(\frac{1}{2}\mathcal{R}^{j_2j_3}_{i_2i_3} - t^2 K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots\right.$$
$$\left.\cdots\times\left(\frac{1}{2}\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}}K^{j_{2n-1}}_{i_{2n-1}}\right) + \frac{(-1)^n}{\ell^{2n-2}}\delta^{j_2}_{i_2}\cdots\delta^{j_{2n-1}}_{i_{2n-1}}\right].$$

Higher even dimensions

EH-AdS gravity +KTs

$$\mathrm{B}_{2n-1} = 2n\sqrt{-h}\int\limits_0^1 dt\;\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2j_3}_{i_2i_3} - t^2K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots$$
$$\cdots\times\left(\frac{1}{2}\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - t^2K^{j_{2n-2}}_{i_{2n-2}}K^{j_{2n-1}}_{i_{2n-1}}\right).$$

$$\tilde{I}_{\mathrm{ren}} = I_{Dir} + \int\limits_{\partial M} d^{2n-1}x\,L_{ct}$$

$$L_{ct} = c_{2n-1}B_{2n-1} + \frac{1}{8\pi G}\sqrt{-h}K$$
$$= \frac{(-\ell^2)^n}{8\pi G(2n-2)!}\sqrt{-h}\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}K^{j_1}_{i_1}\int\limits_0^1 dt\left[\left(\frac{1}{2}\mathcal{R}^{j_2j_3}_{i_2i_3} - t^2K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots\right.$$
$$\left.\cdots\times\left(\frac{1}{2}\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - t^2K^{j_{2n-2}}_{i_{2n-2}}K^{j_{2n-1}}_{i_{2n-1}}\right) + \frac{(-1)^n}{\ell^{2n-2}}\delta^{j_2}_{i_2}\cdots\delta^{j_{2n-1}}_{i_{2n-1}}\right].$$

Higher even dimensions

Expanding and collecting...

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \right.$$
$$\left. + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(2\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n+1)}{4(2n-2)}\mathcal{R}^2 - \frac{(2n-3)}{4}\mathcal{R}^{ijkl}\mathcal{R}_{ijkl}\right) + \cdots\right].$$

Boundary Weyl tensor $\mathcal{W}^{ijkl}\mathcal{W}_{ijkl}$ implies

$$\mathcal{R}^{ijkl}\mathcal{R}_{ijkl} = \mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \frac{4}{(2n-3)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{1}{2(2n-2)}\mathcal{R}^2\right)$$

And, finally

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n-1)}{4(2n-2)}\mathcal{R}^2\right)\right.$$
$$\left. - \frac{\ell^3}{8(2n-3)(2n-5)}\mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) + \cdots\right].$$

Higher even dimensions

Expanding and collecting...

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \right.$$
$$\left. + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(2\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n+1)}{4(2n-2)}\mathcal{R}^2 - \frac{(2n-3)}{4}\mathcal{R}^{ijkl}\mathcal{R}_{ijkl}\right) + \cdots\right].$$

Boundary Weyl tensor $\mathcal{W}^{ijkl}\mathcal{W}_{ijkl}$ implies

$$\mathcal{R}^{ijkl}\mathcal{R}_{ijkl} = \mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \frac{4}{(2n-3)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{1}{2(2n-2)}\mathcal{R}^2\right)$$

And finally

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n-1)}{4(2n-2)}\mathcal{R}^2\right)\right.$$
$$\left. - \frac{\ell^3}{8(2n-3)(2n-5)}\mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) + \cdots\right].$$

Higher even dimensions

Expanding and collecting...

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\Bigg[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \\ + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(2\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n+1)}{4(2n-2)}\mathcal{R}^2 - \frac{(2n-3)}{4}\mathcal{R}^{ijkl}\mathcal{R}_{ijkl}\right) + \cdots\Bigg].$$

Boundary Weyl tensor $\mathcal{W}^{ijkl}\mathcal{W}_{ijkl}$ implies

$$\mathcal{R}^{ijkl}\mathcal{R}_{ijkl} = \mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \frac{4}{(2n-3)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{1}{2(2n-2)}\mathcal{R}^2\right)$$

Higher even dimensions

Expanding and collecting...

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \right.$$
$$\left. + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(2\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n+1)}{4(2n-2)}\mathcal{R}^2 - \frac{(2n-3)}{4}\mathcal{R}^{ijkl}\mathcal{R}_{ijkl}\right) + \cdots\right].$$

Boundary Weyl tensor $\mathcal{W}^{ijkl}\mathcal{W}_{ijkl}$ implies

$$\mathcal{R}^{ijkl}\mathcal{R}_{ijkl} = \mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \frac{4}{(2n-3)}(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{1}{2(2n-2)}\mathcal{R}^2)$$

And, finally...

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n-1)}{4(2n-2)}\mathcal{R}^2\right)\right.$$
$$\left. - \frac{\ell^3}{8(2n-3)(2n-5)}\mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) + \cdots\right].$$

Higher even dimensions

Expanding and collecting...

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\Bigg[\frac{(2n{-}2)}{\ell} + \frac{\ell}{2(2n{-}3)}\mathcal{R} + \\ + \frac{\ell^3}{2(2n{-}3)^2(2n{-}5)}\left(2\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n{+}1)}{4(2n{-}2)}\mathcal{R}^2 - \frac{(2n{-}3)}{4}\mathcal{R}^{ijkl}\mathcal{R}_{ijkl}\right) + \cdots\Bigg].$$

Boundary Weyl tensor $\mathcal{W}^{ijkl}\mathcal{W}_{ijkl}$ implies

$$\mathcal{R}^{ijkl}\mathcal{R}_{ijkl} = \mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \frac{4}{(2n-3)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{1}{2(2n-2)}\mathcal{R}^2\right)$$

And, finally...

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\Bigg[\frac{(2n{-}2)}{\ell} + \frac{\ell}{2(2n{-}3)}\mathcal{R} + \frac{\ell^3}{2(2n{-}3)^2(2n{-}5)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n{-}1)}{4(2n{-}2)}\mathcal{R}^2\right) \\ - \frac{\ell^3}{8(2n{-}3)(2n{-}5)}\mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) + \cdots\Bigg].$$

Holographic Renormalization = Kounterterms?

Well...almost. [G.Anastasiou, O.Miskovic, R.O. and I.Papadimitriou, 2003.06425]

$$\tilde{I}_{\text{ren}} = I_{\text{HR}} - \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int\limits_{\partial M} \sqrt{-h}\,\mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \ldots$$

A similar result in $D = 2n+1$ dimensions.

Last term is zero for most AAdS spaces, but not for gravitational instantons.

Patching up the theory

$$I_{\text{HR}} = \tilde{I}_{\text{ren}} + \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int\limits_{\partial M} \sqrt{-h}\,\mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \ldots$$

Holographic Renormalization = Kounterterms?

Well...almost. [G.Anastasiou, O.Miskovic, R.O. and I.Papadimitriou, 2003.06425]

$$\tilde{I}_{\text{ren}} = I_{\text{HR}} - \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int_{\partial M} \sqrt{-h}\mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \dots$$

A similar result in $D = 2n+1$ dimensions.
Last term is zero for most AAdS spaces, but not for gravitational instantons.

Patching up the theory

$$I_{\text{HR}} = \tilde{I}_{\text{ren}} + \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int_{\partial M} \sqrt{-h}\mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \dots$$

Holographic Renormalization = Kounterterms?

Well...almost. [G.Anastasiou, O.Miskovic, R.O. and I.Papadimitriou, 2003.06425]

$$\tilde{I}_{\text{ren}} = I_{\text{HR}} - \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int\limits_{\partial M} \sqrt{-h}\mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \ldots$$

A similar result in $D = 2n+1$ dimensions.
Last term is zero for most AAdS spaces, but not for gravitational instantons.

Patching up the theory

$$I_{\text{HR}} = \tilde{I}_{\text{ren}} + \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int\limits_{\partial M} \sqrt{-h}\,\mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \ldots$$

Holographic Renormalization = Kounterterms?

Well...almost. [G.Anastasiou, O.Miskovic, R.O. and I.Papadimitriou, 2003.06425]

$$\tilde{I}_{\text{ren}} = I_{\text{HR}} - \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int\limits_{\partial M} \sqrt{-h}\, \mathcal{W}^{ijkl} \mathcal{W}_{ijkl} + \ldots$$

A similar result in $D = 2n+1$ dimensions.

Last term is zero for most AAdS spaces, but not for gravitational instantons.

Patching up the theory

$$I_{\text{HR}} = \tilde{I}_{\text{ren}} + \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int\limits_{\partial M} \sqrt{-h}\, \mathcal{W}^{ijkl} \mathcal{W}_{ijkl} + \ldots$$

Renormalized AdS Action

Euler-Gauss-Bonnet Theorem in 4D

$$\int_{\partial M} d^3x\, B_3(K,\mathcal{R}) = \int_{M} d^4x\, GB - 32\pi^2\chi(M)$$

4D Renormalized AdS action [R. Aros et al., gr-qc/9909015]

$$I_{ren} = \frac{1}{16\pi G}\int_{M} d^4x\sqrt{-g}\left[(R-2\Lambda) + \frac{\ell^2}{4}\left(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)\right]$$

Renormalized AdS Action

Euler-Gauss-Bonnet Theorem in 4D

$$\int_{\partial M} d^3x\, B_3(K,\mathcal{R}) = \int_{M} d^4x\, GB - 32\pi^2\chi(M)$$

4D Renormalized AdS action [R. Aros et al, gr-qc/9909015]

$$I_{\text{ren}} = \frac{1}{16\pi G}\int_{M} d^4x\sqrt{-g}\left[(R-2\Lambda) + \frac{\ell^2}{4}\left(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)\right]$$

Renormalized AdS Action

Euler-Gauss-Bonnet Theorem in 4D

$$\int_{\partial M} d^3x\, B_3(K,\mathcal{R}) = \int_M d^4x\, GB - 32\pi^2\chi(M)$$

4D Renormalized AdS action [R. Aros et al, gr-qc/9909015]

$$I_{\text{ren}} = \frac{1}{16\pi G}\int_M d^4x\sqrt{-g}\left[(R-2\Lambda) + \frac{\ell^2}{4}(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2)\right]$$

Renormalized AdS Action

Euler-Gauss-Bonnet Theorem in 4D

$$\int_{\partial M} d^3x\, B_3(K,\mathcal{R}) = \int_{M} d^4x\, GB - 32\pi^2\chi(M)$$

4D Renormalized AdS action [R. Aros et al, gr-qc/9909015]

$$I_{\text{ren}} = \frac{1}{16\pi G}\int_{M} d^4x\sqrt{-g}\left[(R-2\Lambda) + \frac{\ell^2}{4}\left(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)\right]$$

Renormalized AdS Action

GB coupling is also singled out by SUSY [Andrianopoli and D'Auria, arXiv:1405.2010]

Renormalized AdS action = MacDowell-Mansouri action (1977)

$$I_{\rm ren} = \frac{\ell^2}{256\pi G}\int\limits_M d^4x\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_4]}_{[\mu_1\cdots\mu_4]}\left[R^{\mu_1\mu_2}_{\nu_1\nu_2}+\frac{\delta^{\mu_1\mu_2}_{\nu_1\nu_2}}{\ell^2}\right]\left[R^{\mu_3\mu_4}_{\nu_3\nu_4}+\frac{\delta^{\mu_3\mu_4}_{\nu_3\nu_4}}{\ell^2}\right].$$

Weyl tensor

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - 4S^{[\alpha}_{[\mu}\delta^{\beta]}_{\nu]}, \qquad \text{Schouten } S^\alpha_\mu = \frac{1}{D-2}(R^\alpha_\mu - \frac{1}{2(D-1)}\delta^\alpha_\mu R)$$

Renormalized AdS Action

GB coupling is also singled out by SUSY [Andrianopoli and D'Auria, arXiv:1405.2010]

Renormalized AdS action = MacDowell-Mansouri action (1977)

$$I_{\text{ren}} = \frac{\ell^2}{256\pi G} \int\limits_M d^4x \sqrt{-g}\, \delta^{[\nu_1 \cdots \nu_4]}_{[\mu_1 \cdots \mu_4]} \left[R^{\mu_1\mu_2}_{\nu_1\nu_2} + \frac{\delta^{\mu_1\mu_2}_{\nu_1\nu_2}}{\ell^2} \right] \left[R^{\mu_3\mu_4}_{\nu_3\nu_4} + \frac{\delta^{\mu_3\mu_4}_{\nu_3\nu_4}}{\ell^2} \right].$$

Weyl tensor

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - 4 S^{[\alpha}_{[\mu} \delta^{\beta]}_{\nu]}, \qquad \text{Schouten } S^{\alpha}_{\mu} = \frac{1}{D-2}\left(R^{\alpha}_{\mu} - \frac{1}{2(D-1)} \delta^{\alpha}_{\mu} R\right)$$

Renormalized AdS Action

GB coupling is also singled out by SUSY [Andrianopoli and D'Auria, arXiv:1405.2010]

Renormalized AdS action = MacDowell-Mansouri action (1977)

$$I_{\text{ren}} = \frac{\ell^2}{256\pi G}\int\limits_M d^4x\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_4]}_{[\mu_1\cdots\mu_4]}\left[R^{\mu_1\mu_2}_{\nu_1\nu_2}+\frac{\delta^{\mu_1\mu_2}_{\nu_1\nu_2}}{\ell^2}\right]\left[R^{\mu_3\mu_4}_{\nu_3\nu_4}+\frac{\delta^{\mu_3\mu_4}_{\nu_3\nu_4}}{\ell^2}\right],$$

Weyl tensor

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - 4S^{[\alpha}_{[\mu}\delta^{\beta]}_{\nu]}, \qquad \text{Schouten } S^{\alpha}_{\mu} = \frac{1}{D-2}\left(R^{\alpha}_{\mu} - \frac{1}{2(D-1)}\delta^{\alpha}_{\mu}R\right)$$

Renormalized AdS Action

GB coupling is also singled out by SUSY [Andrianopoli and D'Auria, arXiv:1405.2010]

Renormalized AdS action = MacDowell-Mansouri action (1977)

$$I_{\text{ren}} = \frac{\ell^2}{256\pi G}\int\limits_M d^4x\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_4]}_{[\mu_1\cdots\mu_4]}\left[R^{\mu_1\mu_2}_{\nu_1\nu_2}+\frac{\delta^{\mu_1\mu_2}_{\nu_1\nu_2}}{\ell^2}\right]\left[R^{\mu_3\mu_4}_{\nu_3\nu_4}+\frac{\delta^{\mu_3\mu_4}_{\nu_3\nu_4}}{\ell^2}\right],$$

Weyl tensor

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - 4S^{[\alpha}_{[\mu}\delta^{\beta]}_{\nu]}, \qquad \text{Schouten } S^{\alpha}_{\mu} = \frac{1}{D-2}\left(R^{\alpha}_{\mu} - \frac{1}{2(D-1)}\delta^{\alpha}_{\mu}R\right)$$

Renormalized AdS Action

GB coupling is also singled out by SUSY [Andrianopoli and D'Auria, arXiv:1405.2010]

Renormalized AdS action = MacDowell-Mansouri action (1977)

$$I_{\text{ren}} = \frac{\ell^2}{256\pi G} \int\limits_M d^4x \sqrt{-g}\, \delta^{[\nu_1 \cdots \nu_4]}_{[\mu_1 \cdots \mu_4]} \left[R^{\mu_1 \mu_2}_{\nu_1 \nu_2} + \frac{\delta^{\mu_1 \mu_2}_{\nu_1 \nu_2}}{\ell^2} \right] \left[R^{\mu_3 \mu_4}_{\nu_3 \nu_4} + \frac{\delta^{\mu_3 \mu_4}_{\nu_3 \nu_4}}{\ell^2} \right],$$

Weyl tensor

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - 4 S^{[\alpha}_{[\mu} \delta^{\beta]}_{\nu]}, \qquad \text{Schouten } S^{\alpha}_{\mu} = \frac{1}{D-2}\left(R^{\alpha}_{\mu} - \frac{1}{2(D-1)} \delta^{\alpha}_{\mu} R\right)$$

Renormalized AdS Action

GB coupling is also singled out by SUSY [Andrianopoli and D'Auria, arXiv:1405.2010]

Renormalized AdS action = MacDowell-Mansouri action (1977)

$$I_{\text{ren}} = \frac{\ell^2}{256\pi G} \int\limits_M d^4x \sqrt{-g}\, \delta^{[\nu_1 \cdots \nu_4]}_{[\mu_1 \cdots \mu_4]} \left[R^{\mu_1\mu_2}_{\nu_1\nu_2} + \frac{\delta^{\mu_1\mu_2}_{\nu_1\nu_2}}{\ell^2} \right] \left[R^{\mu_3\mu_4}_{\nu_3\nu_4} + \frac{\delta^{\mu_3\mu_4}_{\nu_3\nu_4}}{\ell^2} \right] ,$$

Weyl tensor

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - 4 S^{[\alpha}_{[\mu} \delta^{\beta]}_{\nu]}, \qquad \text{Schouten } S^{\alpha}_{\mu} = \frac{1}{D-2} (R^{\alpha}_{\mu} - \frac{1}{2(D-1)} \delta^{\alpha}_{\mu} R)$$

Renormalized AdS Action

Weyl tensor for Einstein spaces $S^{\alpha}_{\mu} = -\frac{1}{2\ell^2}\delta^{\alpha}_{\mu}$

$$W^{\alpha\beta}_{(E)\mu\nu} = R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\,\delta^{[\alpha\beta]}_{[\mu\nu]}$$

Renormalized action for Einstein spaces

$$I_{\text{ren}} = \frac{\ell^2}{64\pi G}\int\limits_{M} d^4x\,\sqrt{-g}\,W_{(E)\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}_{(E)}$$

Renormalized AdS Action

Weyl tensor for Einstein spaces $S^{\alpha}_{\mu} = -\frac{1}{2\ell^2}\delta^{\alpha}_{\mu}$

$$W^{\alpha\beta}_{(E)\mu\nu} = R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\,\delta^{[\alpha\beta]}_{[\mu\nu]}$$

Renormalized action for Einstein spaces

$$I_{\mathrm{ren}} = \frac{\ell^2}{64\pi G}\int\limits_M d^4x\,\sqrt{-g}\,W_{(E)\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}_{(E)}$$

Renormalized AdS Action

Weyl tensor for Einstein spaces $S_{\mu}^{\alpha} = -\frac{1}{2\ell^2}\delta_{\mu}^{\alpha}$

$$W_{(E)\mu\nu}^{\alpha\beta} = R_{\mu\nu}^{\alpha\beta} + \frac{1}{\ell^2}\,\delta_{[\mu\nu]}^{[\alpha\beta]}$$

Renormalized action for Einstein spaces

$$I_{\text{ren}} = \frac{\ell^2}{64\pi G}\int\limits_M d^4x\,\sqrt{-g}\,W_{(E)\mu\nu\alpha\beta}W_{(E)}^{\mu\nu\alpha\beta}$$

Renormalized AdS Action

Weyl tensor for Einstein spaces $S_{\mu}^{\alpha} = -\frac{1}{2\ell^2}\delta_{\mu}^{\alpha}$

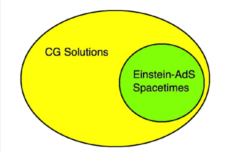
$$W_{(E)\mu\nu}^{\alpha\beta} = R_{\mu\nu}^{\alpha\beta} + \frac{1}{\ell^2}\,\delta_{[\mu\nu]}^{[\alpha\beta]}$$

Renormalized action for Einstein spaces

$$I_{\text{ren}} = \frac{\ell^2}{64\pi G}\int\limits_{M} d^4x\,\sqrt{-g}\,W_{(E)\mu\nu\alpha\beta}W_{(E)}^{\mu\nu\alpha\beta}$$

Conformal Renormalization

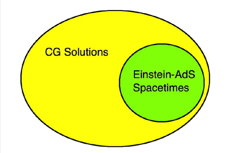
Embedding Einstein theory in Conformal Gravity



- Why?: Conformal Gravity is finite for AAdS conditions. [Grumiller et al., 2013]
- What for?: Renormalization should be inherited by the Einstein sector.
- How?: a *holographic* mechanism to turn CG into Einstein

Conformal Renormalization

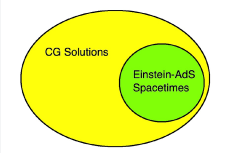
Embedding Einstein theory in Conformal Gravity



- Why?: Conformal Gravity is finite for AAdS conditions. [Grumiller et al., 2013]
- What for?: Renormalization should be inherited by the Einstein sector.
- How?: a *holographic* mechanism to turn CG into Einstein

Conformal Renormalization

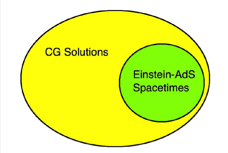
Embedding Einstein theory in Conformal Gravity



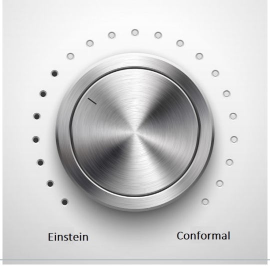
- Why?: Conformal Gravity is finite for AAdS conditions. [Grumiller et al., 2013]
- What for?: Renormalization should be inherited by the Einstein sector.
- How?: a *holographic* mechanism to turn CG into Einstein

Conformal Renormalization

Embedding Einstein theory in Conformal Gravity



- Why?: Conformal Gravity is finite for AAdS conditions. [Grumiller et al., 2013]
- What for?: Renormalization should be inherited by the Einstein sector.
- How?: a *holographic* mechanism to turn CG into Einstein



Einstein Gravity from Conformal Gravity in 4D

Einstein gravity from CG with Neumann bc's [Maldacena, 2011]

$$I_{CG} = \alpha_{CG} \int\limits_M d^4x \sqrt{-g}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

Fefferman-Graham expansion for AAdS spaces in CG

$$ds^2 = \frac{\ell^2}{z^2}\, dz^2 + \frac{1}{z^2}\, g_{ij}(x,z)\, dx^i dx^j\,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots + z g_{(1)ij}(x) + \cdots$$

EOM for Conformal Gravity

$$B_{\mu\nu} = \nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\sigma\nu} = 0\,, \qquad C^\mu_{\nu\lambda} = \nabla_\nu S^\mu_\lambda - \nabla_\lambda S^\mu_\nu$$

Einstein Gravity from Conformal Gravity in 4D

Einstein gravity from CG with Neumann bc's [Maldacena, 2011]

$$I_{CG} = \alpha_{CG} \int_M d^4x \sqrt{-g}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

Fefferman-Graham expansion for AAdS spaces in CG

$$ds^2 = \frac{\ell^2}{z^2}\, dz^2 + \frac{1}{z^2}\, g_{ij}(x,z)\, dx^i dx^j \,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots + z g_{(1)ij}(x) + \cdots$$

EOM for Conformal Gravity

$$B_{\mu\nu} = \nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\sigma\nu} = 0 \,, \qquad C^\mu_{\ \nu\lambda} = \nabla_\nu S^\mu_\lambda - \nabla_\lambda S^\mu_\nu$$

Einstein Gravity from Conformal Gravity in 4D

Einstein gravity from CG with Neumann bc's [Maldacena, 2011]

$$I_{CG} = \alpha_{CG} \int\limits_M d^4x \sqrt{-g}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

Fefferman-Graham expansion for AAdS spaces in CG

$$ds^2 = \frac{\ell^2}{z^2}\, dz^2 + \frac{1}{z^2}\, g_{ij}(x,z)\, dx^i dx^j \,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots + z g_{(1)ij}(x) + \cdots$$

EOM for Conformal Gravity

$$B_{\mu\nu} = \nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\sigma\nu} = 0 \,, \qquad C^\mu_{\nu\lambda} = \nabla_\nu S^\mu_\lambda - \nabla_\lambda S^\mu_\nu$$

Einstein Gravity from Conformal Gravity in 4D

Einstein gravity from CG with Neumann bc's [Maldacena, 2011]

$$I_{CG} = \alpha_{CG} \int\limits_{M} d^4x \sqrt{-g}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

Fefferman-Graham expansion for AAdS spaces in CG

$$ds^2 = \frac{\ell^2}{z^2} dz^2 + \frac{1}{z^2} g_{ij}(x,z)\, dx^i dx^j \,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots + z g_{(1)ij}(x) + \cdots$$

EOM for Conformal Gravity

$$B_{\mu\nu} = \nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\sigma\nu} = 0 \,, \qquad C^{\mu}_{\nu\lambda} = \nabla_\nu S^\mu_\lambda - \nabla_\lambda S^\mu_\nu$$

Einstein Gravity from Conformal Gravity in 4D

Einstein gravity from CG with Neumann bc's [Maldacena, 2011]

$$I_{CG} = \alpha_{CG} \int\limits_M d^4x \sqrt{-g}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

Fefferman-Graham expansion for AAdS spaces in CG

$$ds^2 = \frac{\ell^2}{z^2}\, dz^2 + \frac{1}{z^2}\, g_{ij}(x,z)\, dx^i dx^j \,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots$$
$$+ z g_{(1)ij}(x) + \cdots$$

EOM for Conformal Gravity

$$B_{\mu\nu} = \nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\sigma\nu} = 0 \,, \qquad C^\mu_{\ \nu\lambda} = \nabla_\nu S^\mu_\lambda - \nabla_\lambda S^\mu_\nu$$

Einstein Gravity from Conformal Gravity in 4D

Einstein gravity from CG with Neumann bc's [Maldacena, 2011]

$$I_{CG} = \alpha_{CG} \int_M d^4x \sqrt{-g}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

Fefferman-Graham expansion for AAdS spaces in CG

$$ds^2 = \frac{\ell^2}{z^2}\, dz^2 + \frac{1}{z^2}\, g_{ij}(x,z)\, dx^i dx^j\,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots + z g_{(1)ij}(x) + \cdots$$

EOM for Conformal Gravity

$$B_{\mu\nu} = \nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\sigma\nu} = 0\,, \qquad C^\mu_{\ \nu\lambda} = \nabla_\nu S^\mu_\lambda - \nabla_\lambda S^\mu_\nu$$

Einstein spaces: holographic prescription

Einstein-AdS spaces

$$S^{\mu}_{\nu} = -\frac{1}{2\ell^2}\delta^{\mu}_{\nu}, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Traceless Ricci tensor

$$H^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{D}R\delta^{\mu}_{\nu} = 0$$

$H_{\mu\nu} = 0 \Longleftrightarrow \partial_z g_{ij} = g_{(1)ij} = 0$ and $tr(\partial_z^3 g_{ij}) \sim tr(g_{(3)ij}) = 0$
$g_{(2)ij}$ is free data in CG $\Longrightarrow$chosen as in Einstein $g_{(2)ij} = -\ell^2 S_{(0)ij}$
[Imbimbo, Schwimmer, Theisen and Yankielowicz, hep-th/9910267]

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{\text{CG}}[E] = I_{\text{HR}}$$

Einstein spaces: holographic prescription

Einstein-AdS spaces

$$S^\mu_\nu = -\frac{1}{2\ell^2}\delta^\mu_\nu, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Traceless Ricci tensor

$$H^\mu_\nu = R^\mu_\nu - \frac{1}{D}R\delta^\mu_\nu = 0$$

$H_{\mu\nu} = 0 \Longleftrightarrow \partial_z g_{ij} = g_{(1)ij} = 0$ and $tr(\partial_z^3 g_{ij}) \sim tr(g_{(3)ij}) = 0$
$g_{(2)ij}$ is free data in CG $\Longrightarrow$chosen as in Einstein $g_{(2)ij} = -\ell^2 S_{(0)ij}$
[Imbimbo, Schwimmer, Theisen and Yankielowicz, hep-th/9910267]

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{\text{CG}}[E] = I_{\text{HR}}$$

Einstein spaces: holographic prescription

Einstein-AdS spaces

$$S^{\mu}_{\nu} = -\frac{1}{2\ell^2}\delta^{\mu}_{\nu}, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Traceless Ricci tensor

$$H^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{D}R\delta^{\mu}_{\nu} = 0$$

$H_{\mu\nu} = 0 \Longleftrightarrow \partial_z g_{ij} = g_{(1)ij} = 0$ and $tr(\partial_z^3 g_{ij}) \sim tr(g_{(3)ij}) = 0$
$g_{(2)ij}$ is free data in CG $\Longrightarrow$chosen as in Einstein $g_{(2)ij} = -\ell^2 S_{(0)ij}$
[Imbimbo, Schwimmer, Theisen and Yankielowicz, hep-th/9910267]

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{CG}[E] = I_{\mathrm{HR}}$$

Einstein spaces: holographic prescription

Einstein-AdS spaces

$$S^\mu_\nu = -\frac{1}{2\ell^2}\delta^\mu_\nu, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Traceless Ricci tensor

$$H^\mu_\nu = R^\mu_\nu - \frac{1}{D}R\delta^\mu_\nu = 0$$

$H_{\mu\nu} = 0 \iff \partial_z g_{ij} = g_{(1)ij} = 0$ and $tr(\partial_z^3 g_{ij}) \sim tr(g_{(3)ij}) = 0$

$g_{(2)ij}$ is free data in CG $\Longrightarrow$chosen as in Einstein $g_{(2)ij} = -\ell^2 S_{(0)ij}$

[Imbimbo, Schwimmer, Theisen and Yankielowicz, hep-th/9910267]

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{\rm CG}[E] = I_{\rm HR}$$

Einstein spaces: holographic prescription

Einstein-AdS spaces

$$S^{\mu}_{\nu} = -\frac{1}{2\ell^2}\delta^{\mu}_{\nu}, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Traceless Ricci tensor

$$H^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{D}R\delta^{\mu}_{\nu} = 0$$

$H_{\mu\nu} = 0 \iff \partial_z g_{ij} = g_{(1)ij} = 0$ and $tr(\partial_z^3 g_{ij}) \sim tr(g_{(3)ij}) = 0$

$g_{(2)ij}$ is free data in CG $\Longrightarrow$chosen as in Einstein $g_{(2)ij} = -\ell^2 S_{(0)ij}$

[Imbimbo, Schwimmer, Theisen and Yankielowicz, hep-th/9910267]

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{CG}[E] = I_{HR}$$

Einstein spaces: holographic prescription

Einstein-AdS spaces

$$S^{\mu}_{\nu} = -\frac{1}{2\ell^2}\delta^{\mu}_{\nu}, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Traceless Ricci tensor

$$H^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{D}R\delta^{\mu}_{\nu} = 0$$

$H_{\mu\nu} = 0 \Longleftrightarrow \partial_z g_{ij} = g_{(1)ij} = 0$ and $tr(\partial_z^3 g_{ij}) \sim tr(g_{(3)ij}) = 0$
$g_{(2)ij}$ is free data in CG $\Longrightarrow$chosen as in Einstein $g_{(2)ij} = -\ell^2 S_{(0)ij}$
[Imbimbo, Schwimmer, Theisen and Yankielowicz, hep-th/9910267]

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{CG}[E] = I_{HR}$$

Einstein spaces: holographic prescription

HOLOGRAPHYCL

Einstein-AdS spaces

$$S^\mu_\nu = -\frac{1}{2\ell^2}\delta^\mu_\nu, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Traceless Ricci tensor

$$H^\mu_\nu = R^\mu_\nu - \frac{1}{D}R\delta^\mu_\nu = 0$$

$H_{\mu\nu} = 0 \iff \partial_z g_{ij} = g_{(1)ij} = 0$ and $tr(\partial_z^3 g_{ij}) \sim tr(g_{(3)ij}) = 0$
$g_{(2)ij}$ is free data in CG $\Longrightarrow$chosen as in Einstein $g_{(2)ij} = -\ell^2 S_{(0)ij}$
[Imbimbo, Schwimmer, Theisen and Yankielowicz, hep-th/9910267]

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{\text{CG}}\left[E\right] = I_{\text{HR}}$$

.

Einstein spaces: holographic prescription

Einstein-AdS spaces

$$S^{\mu}_{\nu} = -\frac{1}{2\ell^2}\delta^{\mu}_{\nu}, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Traceless Ricci tensor

$$H^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{D}R\delta^{\mu}_{\nu} = 0$$

$H_{\mu\nu} = 0 \Longleftrightarrow \partial_z g_{ij} = g_{(1)ij} = 0$ and $tr(\partial_z^3 g_{ij}) \sim tr(g_{(3)ij}) = 0$
$g_{(2)ij}$ is free data in CG $\Longrightarrow$chosen as in Einstein $g_{(2)ij} = -\ell^2 S_{(0)ij}$
[Imbimbo, Schwimmer, Theisen and Yankielowicz, hep-th/9910267]

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{\mathsf{CG}}\left[E\right] = I_{\mathsf{HR}}$$

.

AdS gravity in 6D

EH Action+Euler term

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G}\int\limits_{M} d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right),$$

In terms of fully-antisymmetric objects

$$\begin{aligned}\tilde{I}_{\text{ren}} &= \frac{1}{16\pi G\times 192}\int\limits_{M} d^6x\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\Big[R^{\mu_1\mu_2}_{\nu_1\nu_2}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]} \\ &\quad + \frac{2}{3\ell^2}\delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]} - \frac{\ell^4}{3}R^{\mu_1\mu_2}_{\nu_1\nu_2}R^{\mu_3\mu_4}_{\nu_3\nu_4}R^{\mu_5\mu_6}_{\nu_5\nu_6}\Big];\end{aligned}$$

AdS gravity in 6D

EH Action+Euler term

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G} \int\limits_M d^6x \sqrt{-g} \left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72} (Euler)_6 \right) ,$$

In terms of fully antisymmetric objects

$$\begin{aligned} \tilde{I}_{\text{ren}} &= \frac{1}{16\pi G \times 192} \int\limits_M d^6x \sqrt{-g}\, \delta^{[\nu_1 \cdots \nu_6]}_{[\mu_1 \cdots \mu_6]} \left[R^{\mu_1 \mu_2}_{\nu_1 \nu_2} \delta^{[\mu_3 \mu_4]}_{[\nu_3 \nu_4]} \delta^{[\mu_5 \mu_6]}_{[\nu_5 \nu_6]} \right. \\ & \left. + \frac{2}{3\ell^2} \delta^{[\mu_1 \mu_2]}_{[\nu_1 \nu_2]} \delta^{[\mu_3 \mu_4]}_{[\nu_3 \nu_4]} \delta^{[\mu_5 \mu_6]}_{[\nu_5 \nu_6]} - \frac{\ell^4}{3} R^{\mu_1 \mu_2}_{\nu_1 \nu_2} R^{\mu_3 \mu_4}_{\nu_3 \nu_4} R^{\mu_5 \mu_6}_{\nu_5 \nu_6} \right] , \end{aligned}$$

AdS gravity in 6D

EH Action+Euler term

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G}\int\limits_M d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right),$$

In terms of fully-antisymmetric objects

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G\times 192}\int\limits_M d^6x\sqrt{-g}\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\left[R^{\mu_1\mu_2}_{\nu_1\nu_2}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]} + \frac{2}{3\ell^2}\delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]} - \frac{\ell^4}{3}R^{\mu_1\mu_2}_{\nu_1\nu_2}R^{\mu_3\mu_4}_{\nu_3\nu_4}R^{\mu_5\mu_6}_{\nu_5\nu_6}\right],$$

AdS gravity in 6D

EH Action+Euler term

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G}\int\limits_{M} d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right),$$

In terms of fully-antisymmetric objects

$$\begin{aligned}\tilde{I}_{\text{ren}} \quad &= \quad \frac{1}{16\pi G\times 192}\int\limits_{M} d^6x\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\Big[R^{\mu_1\mu_2}_{\nu_1\nu_2}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]} \\ &\qquad +\frac{2}{3\ell^2}\delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]} - \frac{\ell^4}{3}R^{\mu_1\mu_2}_{\nu_1\nu_2}R^{\mu_3\mu_4}_{\nu_3\nu_4}R^{\mu_5\mu_6}_{\nu_5\nu_6}\Big],\end{aligned}$$

AdS gravity in 6D

Polynomial of $W_{(E)}$

$$\tilde{I}_{\text{ren}} = \frac{\ell^4}{16\pi G\times 4!}\int\limits_M d^6x\sqrt{-g}\Big[\frac{1}{2\ell^2}\delta^{[\nu_1\cdots\nu_4]}_{[\mu_1\cdots\mu_4]}W^{\mu_1\mu_2}_{(E)\nu_1\nu_2}W^{\mu_3\mu_4}_{(E)\nu_3\nu_4} - \frac{1}{4!}\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}W^{\mu_1\mu_2}_{(E)\nu_1\nu_2}W^{\mu_3\mu_4}_{(E)\nu_3\nu_4}W^{\mu_5\mu_6}_{(E)\nu_5\nu_6}\Big],$$

Conformal Gravity in 6D

There are three Conformal Invariants in 6D

$$
\begin{aligned}
I_1 &= W_{\alpha\beta\mu\nu} W^{\alpha\sigma\lambda\nu} W_{\sigma}{}^{\beta\mu}{}_{\lambda}\,, \\
I_2 &= W_{\mu\nu\alpha\beta} W^{\alpha\beta\sigma\lambda} W_{\sigma\lambda}{}^{\mu\nu}\,, \\
I_3 &= W_{\mu\rho\sigma\lambda} \left(\delta^{\mu}_{\nu} \Box + 4R^{\mu}_{\nu} - \frac{6}{5} R \delta^{\mu}_{\nu} \right) W^{\nu\rho\sigma\lambda} + \nabla_{\mu} J^{\mu}\,,
\end{aligned}
$$

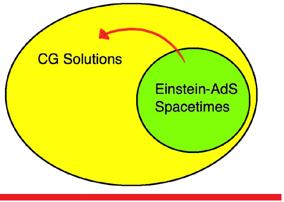
with

$$
\begin{aligned}
J_{\mu} \;\; &= \;\; 4R_{\mu}{}^{\lambda\rho\sigma} \nabla^{\nu} R_{\nu\lambda\rho\sigma} + 3R^{\nu\lambda\rho\sigma} \nabla_{\mu} R_{\nu\lambda\rho\sigma} - 5R^{\nu\lambda} \nabla_{\mu} R_{\nu\lambda} \\
&\quad + \frac{1}{2} R \nabla_{\mu} R - R^{\nu}_{\mu} \nabla_{\nu} R + 2R^{\nu\lambda} \nabla_{\nu} R_{\lambda\mu}.
\end{aligned}
$$

Conformal Covariantization

Einstein	$\to$	**Conformal**	**CI's**
$\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\mathbf{W}^{\mu_1\mu_2}_{(E)\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{(E)\nu_3\nu_4}\mathbf{W}^{\mu_5\mu_6}_{(E)\nu_5\nu_6}$	$\to$	$\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\mathbf{W}^{\mu_1\mu_2}_{\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{\nu_3\nu_4}\mathbf{W}^{\mu_5\mu_6}_{\nu_5\nu_6}$	$32(2\mathbf{I}_1+\mathbf{I}_2)$
$-\frac{1}{\ell^2}\delta^{[\nu_1\cdots\nu_4]}_{[\mu_1\cdots\mu_4]}\mathbf{W}^{\mu_1\mu_2}_{(E)\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{(E)\nu_3\nu_4}$	$\to$	$\delta^{[\nu_1\cdots\nu_5]}_{[\mu_1\cdots\mu_5]}\mathbf{W}^{\mu_1\mu_2}_{\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{\nu_3\nu_4}\mathbf{S}^{\mu_5}_{\nu_5}+\mathbf{16C}^{\mu\nu\lambda}\mathbf{C}_{\mu\nu\lambda}+\nabla^{\mu}\mathbf{J}_{\mu}$	$4\mathbf{I}_1-\mathbf{I}_2-\mathbf{I}_3$

$$\mathbf{J}_{\mu}=\mathbf{16W}^{\kappa\lambda\nu}_{\mu}\mathbf{C}_{\kappa\lambda\nu}-\mathbf{2W}^{\kappa\lambda}_{\nu\sigma}\nabla_{\mu}\mathbf{W}^{\nu\sigma}_{\kappa\lambda}$$



Lu-Pang-Pope CG in 6D

6D CG with an Einstein sector [Lu, Pang and Pope, 2013]

$$I_{CG} = \alpha_{CG} \int\limits_M d^6x\sqrt{-\hat{g}}\left(\frac{1}{4!}\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}W^{\mu_1\mu_2}_{\nu_1\nu_2}W^{\mu_3\mu_4}_{\nu_3\nu_4}W^{\mu_5\mu_6}_{\nu_5\nu_6} + \frac{1}{2}\delta^{[\nu_1\cdots\nu_5]}_{[\mu_1\cdots\mu_5]}W^{\mu_1\mu_2}_{\nu_1\nu_2}W^{\mu_3\mu_4}_{\nu_3\nu_4}S^{\mu_5}_{\nu_5}\right.$$
$$\left. + 8C^{\mu\nu\lambda}C_{\mu\nu\lambda}\right) + \alpha_{CG\partial M} d^5x\sqrt{-h}n^\mu\left(8W^{\kappa\lambda\nu}_{\mu}C_{\kappa\lambda\nu} - W^{\kappa\lambda}_{\nu\sigma}\nabla_\mu W^{\nu\sigma}_{\kappa\lambda}\right).$$

- LPP action appears as type-B anomaly and one-loop divergences in 7D
- Variation of I_{CG} gives EOM in terms of Weyl, Cotton and Schouten tensors.
- Any Einstein-AdS spacetime is solution of LPP CG. [Anastasiou, Araya and RO, 2010.15146]

Lu-Pang-Pope CG in 6D

6D CG with an Einstein sector [Lu, Pang and Pope, 2013]

$$I_{CG}=\alpha_{CG}\int\limits_{M}d^6x\sqrt{-\hat{g}}\left(\frac{1}{4!}\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}W^{\mu_1\mu_2}_{\nu_1\nu_2}W^{\mu_3\mu_4}_{\nu_3\nu_4}W^{\mu_5\mu_6}_{\nu_5\nu_6}+\frac{1}{2}\delta^{[\nu_1\cdots\nu_5]}_{[\mu_1\cdots\mu_5]}W^{\mu_1\mu_2}_{\nu_1\nu_2}W^{\mu_3\mu_4}_{\nu_3\nu_4}S^{\mu_5}_{\nu_5}\right.$$
$$\left.+8C^{\mu\nu\lambda}C_{\mu\nu\lambda}\right)+\alpha_{CG\,\partial M}d^5x\sqrt{-h}n^{\mu}\left(8W^{\kappa\lambda\nu}_{\mu}C_{\kappa\lambda\nu}-W^{\kappa\lambda}_{\nu\sigma}\nabla_{\mu}W^{\nu\sigma}_{\kappa\lambda}\right).$$

- LPP action appears as type-B anomaly and one-loop divergences in 7D
- Variation of I_{CG} gives EOM in terms of Weyl, Cotton and Schouten tensors.
- Any Einstein-AdS spacetime is solution of LPP CG. [Anastasiou, Araya and RO, 2010.15146]

Lu-Pang-Pope CG in 6D

6D CG with an Einstein sector [Lu, Pang and Pope, 2013]

$$I_{CG} = \alpha_{CG} \int\limits_{M} d^6x \sqrt{-\hat{g}} \left(\frac{1}{4!} \delta^{[\nu_1 \cdots \nu_6]}_{[\mu_1 \cdots \mu_6]} W^{\mu_1\mu_2}_{\nu_1\nu_2} W^{\mu_3\mu_4}_{\nu_3\nu_4} W^{\mu_5\mu_6}_{\nu_5\nu_6} + \frac{1}{2} \delta^{[\nu_1 \cdots \nu_5]}_{[\mu_1 \cdots \mu_5]} W^{\mu_1\mu_2}_{\nu_1\nu_2} W^{\mu_3\mu_4}_{\nu_3\nu_4} S^{\mu_5}_{\nu_5} \right.$$
$$\left. + 8 C^{\mu\nu\lambda} C_{\mu\nu\lambda} \right) + \alpha_{CG} \partial M \, d^5x \sqrt{-h} n^{\mu} \left(8 W^{\kappa\lambda\nu}_{\mu} C_{\kappa\lambda\nu} - W^{\kappa\lambda}_{\nu\sigma} \nabla_{\mu} W^{\nu\sigma}_{\kappa\lambda} \right).$$

- LPP action appears as type-B anomaly and one-loop divergences in 7D
- Variation of I_{CG} gives EOM in terms of Weyl, Cotton and Schouten tensors.
- Any Einstein-AdS spacetime is solution of LPP CG. [Anastasiou, Araya and RO, 2010.15146]

Lu-Pang-Pope CG in 6D

6D CG with an Einstein sector [Lu, Pang and Pope, 2013]

$$I_{CG}=\alpha_{CG}\int\limits_{M}d^6x\sqrt{-\hat{g}}\left(\frac{1}{4!}\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}W^{\mu_1\mu_2}_{\nu_1\nu_2}W^{\mu_3\mu_4}_{\nu_3\nu_4}W^{\mu_5\mu_6}_{\nu_5\nu_6}+\frac{1}{2}\delta^{[\nu_1\cdots\nu_5]}_{[\mu_1\cdots\mu_5]}W^{\mu_1\mu_2}_{\nu_1\nu_2}W^{\mu_3\mu_4}_{\nu_3\nu_4}S^{\mu_5}_{\nu_5}\right.$$
$$\left.+8C^{\mu\nu\lambda}C_{\mu\nu\lambda}\right)+\alpha_{CG}\,{}_{\partial M}d^5x\sqrt{-h}n^{\mu}\left(8W^{\kappa\lambda\nu}_{\mu}C_{\kappa\lambda\nu}-W^{\kappa\lambda}_{\nu\sigma}\nabla_{\mu}W^{\nu\sigma}_{\kappa\lambda}\right).$$

- LPP action appears as type-B anomaly and one-loop divergences in 7D
- Variation of I_{CG} gives EOM in terms of Weyl, Cotton and Schouten tensors.
- Any Einstein-AdS spacetime is solution of LPP CG. [Anastasiou, Araya and RO, 2010.15146]

Lu-Pang-Pope CG in 6D

6D CG with an Einstein sector [Lu, Pang and Pope, 2013]

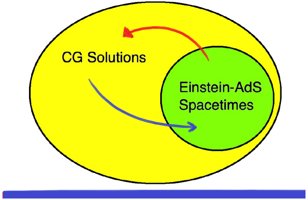
$$I_{CG} = \alpha_{CG} \int_M d^6x \sqrt{-\hat{g}} \left(\frac{1}{4!} \delta^{[\nu_1 \cdots \nu_6]}_{[\mu_1 \cdots \mu_6]} W^{\mu_1\mu_2}_{\nu_1\nu_2} W^{\mu_3\mu_4}_{\nu_3\nu_4} W^{\mu_5\mu_6}_{\nu_5\nu_6} + \frac{1}{2} \delta^{[\nu_1 \cdots \nu_5]}_{[\mu_1 \cdots \mu_5]} W^{\mu_1\mu_2}_{\nu_1\nu_2} W^{\mu_3\mu_4}_{\nu_3\nu_4} S^{\mu_5}_{\nu_5} \right.$$
$$\left. + 8 C^{\mu\nu\lambda} C_{\mu\nu\lambda} \right) + \alpha_{CG} \partial M d^5x \sqrt{-h} n^\mu \left(8 W^{\kappa\lambda\nu}_{\mu} C_{\kappa\lambda\nu} - W^{\kappa\lambda}_{\nu\sigma} \nabla_\mu W^{\nu\sigma}_{\kappa\lambda} \right).$$

- LPP action appears as type-B anomaly and one-loop divergences in 7D
- Variation of I_{CG} gives EOM in terms of Weyl, Cotton and Schouten tensors.
- Any Einstein-AdS spacetime is solution of LPP CG. [Anastasiou, Araya and RO, 2010.15146]

And back...(Einstein gravity from CG in 6D)

LPP CG action decomposed into Einstein and non-Einstein parts:

$$
\begin{aligned}
I_{CG} &= -4!\alpha_{CG}\int\limits_{M} d^6x\sqrt{-g}\left[P_6\left(W_{(E)}\right)+Q\left(W_{(E)},H\right)\right] \\
&\quad -\alpha_{CG}\int\limits_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W^{\kappa\lambda}_{(E)\nu\sigma}\nabla_{\mu}W^{\nu\sigma}_{(E)\kappa\lambda}\right).
\end{aligned}
$$



Back to Einstein gravity (with an extra term)

Einstein condition, and $\alpha_{\mathsf{E}} = -\frac{\ell^4}{384\pi G}$:

$$I_{CG}\left[E\right] = \frac{1}{16\pi G}\int\limits_{M} d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right) + \frac{\ell^4}{384\pi G}\int\limits_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W_{(E)\nu\sigma}^{\kappa\lambda}\nabla_{\mu}W_{(E)\kappa\lambda}^{\nu\sigma}\right),$$

Performing asymptotic expansions

$$\Delta I = \frac{\ell^8}{192\pi G}\int\limits_{\partial M} d^5x\sqrt{-h}W^{ijkl}(h)W_{ijkl}(h) + \ldots$$

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{\mathsf{CG}}\left[E\right] = I_{\mathsf{HR}}$$

Back to Einstein gravity (with an extra term)

Einstein condition, and $\alpha_{\mathrm{E}} = -\frac{\ell^4}{384\pi G}$:

$$I_{CG}\left[E\right] = \frac{1}{16\pi G}\int\limits_{M} d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right) + \frac{\ell^4}{384\pi G}\int\limits_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W^{\kappa\lambda}_{(E)\nu\sigma}\nabla_{\mu}W^{\nu\sigma}_{(E)\kappa\lambda}\right),$$

Performing asymptotic expansions

$$\Delta I = \frac{\ell^8}{192\pi G}\int\limits_{\partial M} d^5x\sqrt{-h}W^{ijkl}(h)W_{ijkl}(h) + \ldots$$

CG action for Einstein spaces = Renormalized Einstein AdS action

$$I_{\mathrm{CG}}\left[E\right] = I_{\mathrm{HR}}$$

Back to Einstein gravity (with an extra term)

Einstein condition, and $\alpha_{\mathrm{E}} = -\frac{\ell^4}{384\pi G}$:

$$I_{CG}\left[E\right] = \frac{1}{16\pi G}\int\limits_{M} d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right) + \frac{\ell^4}{384\pi G}\int\limits_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W_{(E)\nu\sigma}^{\kappa\lambda}\nabla_{\mu}W_{(E)\kappa\lambda}^{\nu\sigma}\right),$$

Performing asymptotic expansions

$$\Delta I = \frac{\ell^3}{192\pi G}\int\limits_{\partial M} d^5x\sqrt{-h}\mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) + \ldots$$

CG action for Einstein spaces = Renormalized Einstein AdS action

$$I_{\mathrm{CG}}\left[E\right] = I_{\mathrm{HR}}$$

Back to Einstein gravity (with an extra term)

Einstein condition, and $\alpha_{\mathsf{E}} = -\frac{\ell^4}{384\pi G}$:

$$I_{CG}[E] = \frac{1}{16\pi G}\int_M d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right) + \frac{\ell^4}{384\pi G}\int_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W_{(E)}^{\kappa\lambda}{}_{\nu\sigma}\nabla_{\mu}W_{(E)}^{\nu\sigma}{}_{\kappa\lambda}\right),$$

Performing asymptotic expansions

$$\Delta I = \frac{\ell^3}{192\pi G}\int_{\partial M} d^5x\sqrt{-h}\mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) + \ldots$$

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{CG}[E] = I_{\mathsf{HR}}$$

Back to Einstein gravity (with an extra term)

Einstein condition, and $\alpha_{\mathrm{E}} = -\frac{\ell^4}{384\pi G}$:

$$I_{CG}[E] = \frac{1}{16\pi G}\int_M d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right) + \frac{\ell^4}{384\pi G}\int_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W_{(E)\nu\sigma}^{\kappa\lambda}\nabla_{\mu}W_{(E)\kappa\lambda}^{\nu\sigma}\right),$$

Performing asymptotic expansions

$$\Delta I = \frac{\ell^3}{192\pi G}\int_{\partial M} d^5x\sqrt{-h}\mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) + \ldots$$

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{\mathrm{CG}}[E] = I_{\mathrm{HR}}$$

.

Back to Einstein gravity (with an extra term)

Einstein condition, and $\alpha_{\mathsf{E}} = -\frac{\ell^4}{384\pi G}$:

$$I_{CG}\left[E\right] = \frac{1}{16\pi G}\int\limits_{M} d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right) + \frac{\ell^4}{384\pi G}\int\limits_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W_{(E)\nu\sigma}^{\kappa\lambda}\nabla_{\mu}W_{(E)\kappa\lambda}^{\nu\sigma}\right),$$

Performing asymptotic expansions

$$\Delta I = \frac{\ell^3}{192\pi G}\int\limits_{\partial M} d^5x\sqrt{-h}\mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) + \ldots$$

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{\mathsf{CG}}\left[E\right] = I_{\mathsf{HR}}$$

.

Curvature-squared terms and conical defects

Riemann squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Rie^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Rie^2 + 8\pi\,(1-\alpha)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{ABAB} - \mathcal{K}^{(A)}_{mn}\mathcal{K}^{mn}_{(A)}\right) + \ldots$$

Ricci squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Ric^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Ric^2 + 4\pi\,(1-\alpha)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{AA} - \frac{1}{2}\mathcal{K}^{(A)}\mathcal{K}_{(A)}\right) + \ldots$$

Ricci scalar squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(R^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,R^2 + 8\pi\,(1-\alpha)\int_\Sigma \sqrt{\gamma}\,R + \ldots$$

[Fursaev-Patrushev-Solodukhin, 2013]

Curvature-squared terms and conical defects

Riemann squared term

$$\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}\left(Rie^{(\alpha)}\right)^2 = \int\limits_{M} d^4x\sqrt{g}\,Rie^2 + 8\pi\left(1-\alpha\right)\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left(R_{ABAB} - \mathcal{K}^{(A)}_{mn}\mathcal{K}^{mn}_{(A)}\right) + \ldots$$

Ricci squared term

$$\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}\left(Ric^{(\alpha)}\right)^2 = \int\limits_{M} d^4x\sqrt{g}\,Ric^2 + 4\pi\left(1-\alpha\right)\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left(R_{AA} - \frac{1}{2}\mathcal{K}^{(A)}\mathcal{K}_{(A)}\right) + \ldots$$

Ricci scalar squared term

$$\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}\left(R^{(\alpha)}\right)^2 = \int\limits_{M} d^4x\sqrt{g}\,R^2 + 8\pi\left(1-\alpha\right)\int\limits_{\Sigma}\sqrt{\gamma}\,R + \ldots$$

[Fursaev-Patrushev-Solodukhin, 2013]

Curvature-squared terms and conical defects

Riemann squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Rie^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Rie^2 + 8\pi\left(1-\alpha\right)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{ABAB} - \mathcal{K}^{(A)}_{mn}\mathcal{K}^{mn}_{(A)}\right) + \ldots$$

Ricci squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Ric^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Ric^2 + 4\pi\left(1-\alpha\right)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{AA} - \frac{1}{2}\mathcal{K}^{(A)}\mathcal{K}_{(A)}\right) + \ldots$$

Ricci scalar squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(R^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,R^2 + 8\pi\left(1-\alpha\right)\int_\Sigma \sqrt{\gamma}\,R + \ldots$$

[Fursaev-Patrushev-Solodukhin, 2013]

Curvature-squared terms and conical defects

Riemann squared term

$$\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}\left(Rie^{(\alpha)}\right)^2 = \int\limits_{M} d^4x\sqrt{g}\,Rie^2 + 8\pi\left(1-\alpha\right)\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left(R_{ABAB} - \mathcal{K}^{(A)}_{mn}\mathcal{K}^{mn}_{(A)}\right) + \ldots$$

Ricci squared term

$$\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}\left(Ric^{(\alpha)}\right)^2 = \int\limits_{M} d^4x\sqrt{g}\,Ric^2 + 4\pi\left(1-\alpha\right)\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left(R_{AA} - \frac{1}{2}\mathcal{K}^{(A)}\mathcal{K}_{(A)}\right) + \ldots$$

Ricci scalar squared term

$$\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}\left(R^{(\alpha)}\right)^2 = \int\limits_{M} d^4x\sqrt{g}\,R^2 + 8\pi\left(1-\alpha\right)\int\limits_{\Sigma}\sqrt{\gamma}\,R + \ldots$$

[Fursaev-Patrushev-Solodukhin, 2013]

Curvature-squared terms and conical defects

Riemann squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Rie^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Rie^2 + 8\pi(1-\alpha)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{ABAB} - \mathcal{K}^{(A)}_{mn}\mathcal{K}^{mn}_{(A)}\right) + \ldots$$

Ricci squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Ric^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Ric^2 + 4\pi(1-\alpha)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{AA} - \frac{1}{2}\mathcal{K}^{(A)}\mathcal{K}_{(A)}\right) + \ldots$$

Ricci scalar squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(R^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,R^2 + 8\pi(1-\alpha)\int_\Sigma \sqrt{\gamma}\,R + \ldots$$

[Fursaev-Patrushev-Solodukhin, 2013]

Curvature-squared terms and conical defects

Riemann squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Rie^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Rie^2 + 8\pi\left(1-\alpha\right)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{ABAB} - \mathcal{K}^{(A)}_{mn}\mathcal{K}^{mn}_{(A)}\right) + \ldots$$

Ricci squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Ric^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Ric^2 + 4\pi\left(1-\alpha\right)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{AA} - \frac{1}{2}\mathcal{K}^{(A)}\mathcal{K}_{(A)}\right) + \ldots$$

Ricci scalar squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(R^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,R^2 + 8\pi\left(1-\alpha\right)\int_\Sigma \sqrt{\gamma}\,R + \ldots$$

[Fursaev-Patrushev-Solodukhin, 2013]

Curvature-squared terms and conical defects

Riemann squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Rie^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Rie^2 + 8\pi\left(1-\alpha\right)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{ABAB} - \mathcal{K}^{(A)}_{mn}\mathcal{K}^{mn}_{(A)}\right) + \ldots$$

Ricci squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Ric^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Ric^2 + 4\pi\left(1-\alpha\right)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{AA} - \frac{1}{2}\mathcal{K}^{(A)}\mathcal{K}_{(A)}\right) + \ldots$$

Ricci scalar squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(R^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,R^2 + 8\pi\left(1-\alpha\right)\int_\Sigma \sqrt{\gamma}\,R + \ldots$$

[Fursaev-Patrushev-Solodukhin, 2013]

Conformal Gravity and Conical Defects

4D Conformal Gravity

$$I_{CG} = \frac{\ell^2}{64\pi G}\int\limits_M d^4x\,\sqrt{-g}\,W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta} - \frac{\pi\ell^2}{2G}\chi\left[M\right]$$

4D Conformal Gravity

$$I_{CG} = \frac{\ell^2}{64\pi G}\int\limits_M d^4x\sqrt{g}\left(Ric^2 - 2Ric^2 + \frac{1}{3}R^2\right) - \frac{\pi\ell^2}{2G}\chi\left[M\right]$$

In a manifold with a conical singularity

$$\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}\left|W^{(\alpha)}\right|^2 = \int\limits_M d^4x\sqrt{g}\left|W\right|^2 + 8\pi\left(1-\alpha\right)\int\limits_\Sigma d^2y\sqrt{\gamma}K_\Sigma + O\left(\left(1-\alpha\right)^2\right)$$

Conformal Gravity and Conical Defects

4D Conformal Gravity

$$I_{CG} = \frac{\ell^2}{64\pi G}\int\limits_M d^4x\,\sqrt{-g}\,W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta} - \frac{\pi\ell^2}{2G}\chi\left[M\right]$$

Conformal Gravity and Conical Defects

4D Conformal Gravity

$$I_{CG} = \frac{\ell^2}{64\pi G} \int\limits_M d^4x \sqrt{-g}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta} - \frac{\pi\ell^2}{2G}\chi\left[M\right]$$

4D Conformal Gravity

$$I_{\mathrm{CG}} = \frac{\ell^2}{64\pi G} \int\limits_M d^4x \sqrt{g} \left(Rie^2 - 2Ric^2 + \frac{1}{3}R^2 \right) - \frac{\pi\ell^2}{2G}\chi\left[M\right]$$

In a manifold with a conical singularity

$$\int\limits_{M^{(\alpha)}} d^4x \sqrt{g} \left| W^{(\alpha)} \right|^2 = \int\limits_M d^4x \sqrt{g} \left| W \right|^2 + 8\pi(1-\alpha) \int\limits_\Sigma d^2y \sqrt{\gamma} K_\Sigma + O\left((1-\alpha)^2 \right)$$

Conformal Gravity and Conical Defects

4D Conformal Gravity

$$I_{CG}=\frac{\ell^2}{64\pi G}\int\limits_M d^4x\,\sqrt{-g}\,W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}-\frac{\pi\ell^2}{2G}\chi\left[M\right]$$

4D Conformal Gravity

$$I_{\mathsf{CG}}=\frac{\ell^2}{64\pi G}\int\limits_M d^4x\sqrt{g}\left(Rie^2-2Ric^2+\frac{1}{3}R^2\right)-\frac{\pi\ell^2}{2G}\chi\left[M\right]$$

In a manifold with a conical singularity

$$\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}\left|W^{(\alpha)}\right|^2=\int\limits_M d^4x\sqrt{g}\left|W\right|^2+8\pi\left(1-\alpha\right)\int\limits_\Sigma d^2y\sqrt{\gamma}K_\Sigma+O\left(\left(1-\alpha\right)^2\right)$$

Conformal Gravity and Conical Defects

4D Conformal Gravity

$$I_{CG} = \frac{\ell^2}{64\pi G} \int\limits_M d^4x\, \sqrt{-g}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta} - \frac{\pi \ell^2}{2G} \chi\left[M\right]$$

4D Conformal Gravity

$$I_{\mathrm{CG}} = \frac{\ell^2}{64\pi G} \int\limits_M d^4x \sqrt{g} \left(Rie^2 - 2Ric^2 + \frac{1}{3} R^2 \right) - \frac{\pi \ell^2}{2G} \chi\left[M\right]$$

In a manifold with a conical singularity

$$\int\limits_{M^{(\alpha)}} d^4x \sqrt{g} \left| W^{(\alpha)} \right|^2 = \int\limits_M d^4x \sqrt{g} \left| W \right|^2 + 8\pi \left(1-\alpha\right) \int\limits_\Sigma d^2y \sqrt{\gamma} K_\Sigma + \mathcal{O}\left(\left(1-\alpha\right)^2 \right)$$

Conformal Gravity and Conical Defects

4D Conformal Gravity

$$I_{CG}=\frac{\ell^2}{64\pi G}\int\limits_M d^4x\,\sqrt{-g}\,W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}-\frac{\pi\ell^2}{2G}\chi\left[M\right]$$

4D Conformal Gravity

$$I_{\mathsf{CG}}=\frac{\ell^2}{64\pi G}\int\limits_M d^4x\sqrt{g}\left(Rie^2-2Ric^2+\frac{1}{3}R^2\right)-\frac{\pi\ell^2}{2G}\chi\left[M\right]$$

In a manifold with a conical singularity

$$\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}\left|W^{(\alpha)}\right|^2=\int\limits_M d^4x\sqrt{g}\left|W\right|^2+8\pi\left(1-\alpha\right)\int\limits_\Sigma d^2y\sqrt{\gamma}K_\Sigma+\mathcal{O}\left(\left(1-\alpha\right)^2\right)$$

Conformal Gravity and Conical Defects

4D Conformal Gravity

$$I_{CG} = \frac{\ell^2}{64\pi G} \int\limits_M d^4x \sqrt{-g}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta} - \frac{\pi\ell^2}{2G} \chi\left[M\right]$$

4D Conformal Gravity

$$I_{\mathsf{CG}} = \frac{\ell^2}{64\pi G} \int\limits_M d^4x \sqrt{g} \left(Rie^2 - 2Ric^2 + \frac{1}{3} R^2 \right) - \frac{\pi\ell^2}{2G} \chi\left[M\right]$$

In a manifold with a conical singularity

$$\int\limits_{M^{(\alpha)}} d^4x \sqrt{g} \left| W^{(\alpha)} \right|^2 = \int\limits_M d^4x \sqrt{g} \left| W \right|^2 + 8\pi \left(1-\alpha\right) \int\limits_\Sigma d^2y \sqrt{\gamma} K_\Sigma + \mathcal{O}\left(\left(1-\alpha\right)^2 \right)$$

Conformal Gravity and Conical Defects

Conformal Invariant in codimension-2 (Graham-Witten anomaly)

$$K_{\Sigma} = W^{mn}_{mn} - P^{(A)}_{mn} P^{mn}_{(A)} ,$$

Traceless part of the extrinsic curvature

$$P^{(A)}_{mn} = \mathcal{K}^{(A)}_{mn} - \frac{1}{2} \mathcal{K}^{(A)} \gamma_{mn}$$

Conformal Gravity Action

$$I^{(\alpha)}_{\mathrm{CG}} = I_{\mathrm{CG}} + \frac{(1-\alpha)}{4G} L_{\Sigma} + \mathcal{O}\left((1-\alpha)^2\right)$$

Conformal Invariance in Codimension-2 is inherited from the Bulk!

Conformal Gravity and Conical Defects

Conformal Invariant in codimension-2 (Graham-Witten anomaly)

$$K_{\Sigma} = W^{mn}_{mn} - P^{(A)}_{mn} P^{mn}_{(A)} \,,$$

Traceless part of the extrinsic curvature

$$P^{(A)}_{mn} = \mathcal{K}^{(A)}_{mn} - \frac{1}{2}\mathcal{K}^{(A)}\gamma_{mn}$$

Conformal Gravity Action

$$I^{(\alpha)}_{\mathrm{CG}} = I_{\mathrm{CG}} + \frac{(1-\alpha)}{4G} L_{\Sigma} + O\left((1-\alpha)^2\right)$$

Conformal Invariance in Codimension-2 is inherited from the Bulk!

Conformal Gravity and Conical Defects

Conformal Invariant in codimension-2 (Graham-Witten anomaly)

$$K_{\Sigma} = W^{mn}_{mn} - P^{(A)}_{mn} P^{mn}_{(A)} \,,$$

Traceless part of the extrinsic curvature

$$P^{(A)}_{mn} = \mathcal{K}^{(A)}_{mn} - \frac{1}{2}\mathcal{K}^{(A)}\gamma_{mn}$$

Conformal Gravity Action

$$I^{(\alpha)}_{\mathrm{CG}} = I_{\mathrm{CG}} + \frac{(1-\alpha)}{4G} L_{\Sigma} + \mathcal{O}\left((1-\alpha)^2\right)$$

Conformal Invariance in Codimension-2 is inherited from the Bulk

Conformal Gravity and Conical Defects

Conformal Invariant in codimension-2 (Graham-Witten anomaly)

$$K_{\Sigma} = W^{mn}_{mn} - P^{(A)}_{mn} P^{mn}_{(A)} \,,$$

Traceless part of the extrinsic curvature

$$P^{(A)}_{mn} = \mathcal{K}^{(A)}_{mn} - \frac{1}{2}\mathcal{K}^{(A)}\gamma_{mn}$$

Conformal Gravity Action

$$I^{(\alpha)}_{\text{CG}} = I_{\text{CG}} + \frac{(1-\alpha)}{4G} L_{\Sigma} + \mathcal{O}\left((1-\alpha)^2\right)$$

Conformal Invariance in Codimension-2 is inherited from the Bulk!

Conformal Gravity and Conical Defects

Conformal Invariant in codimension-2 (Graham-Witten anomaly)

$$K_{\Sigma} = W^{mn}_{mn} - P^{(A)}_{mn} P^{mn}_{(A)} \, ,$$

Traceless part of the extrinsic curvature

$$P^{(A)}_{mn} = \mathcal{K}^{(A)}_{mn} - \frac{1}{2} \mathcal{K}^{(A)} \gamma_{mn}$$

Conformal Gravity Action

$$I^{(\alpha)}_{\mathsf{CG}} = I_{\mathsf{CG}} + \frac{(1-\alpha)}{4G} L_{\Sigma} + \mathcal{O}\left((1-\alpha)^2 \right)$$

Conformal Invariance in Codimension-2 is inherited from the Bulk!

Conformal Gravity and Conical Defects

Conformal Invariant in codimension-2 (Graham-Witten anomaly)

$$K_{\Sigma} = W^{mn}_{mn} - P^{(A)}_{mn} P^{mn}_{(A)} \, ,$$

Traceless part of the extrinsic curvature

$$P^{(A)}_{mn} = \mathcal{K}^{(A)}_{mn} - \frac{1}{2} \mathcal{K}^{(A)} \gamma_{mn}$$

Conformal Gravity Action

$$I^{(\alpha)}_{\mathsf{CG}} = I_{\mathsf{CG}} + \frac{(1-\alpha)}{4G} L_{\Sigma} + \mathcal{O}\left((1-\alpha)^2 \right)$$

Conformal Invariance in Codimension-2 is inherited from the Bulk!

Conformal Gravity and Conical Defects

Conformal Invariant in codimension-2 (Graham-Witten anomaly)

$$K_{\Sigma} = W^{mn}_{mn} - P^{(A)}_{mn} P^{mn}_{(A)} \,,$$

Traceless part of the extrinsic curvature

$$P^{(A)}_{mn} = \mathcal{K}^{(A)}_{mn} - \frac{1}{2}\mathcal{K}^{(A)}\gamma_{mn}$$

Conformal Gravity Action

$$I^{(\alpha)}_{\mathsf{CG}} = I_{\mathsf{CG}} + \frac{(1-\alpha)}{4G} L_{\Sigma} + \mathcal{O}\left((1-\alpha)^2\right)$$

Conformal Invariance in Codimension-2 is inherited from the Bulk!

Renormalized Area

Minimal surface/Einstein ambient space

$$\mathcal{A}_{\text{ren}} = L_{\Sigma_{\text{min}}}[E]$$

Renormalized Area

$$\mathcal{A}_{ren}[\Sigma] = \frac{\ell^2}{2} \int\limits_{\Sigma} d^2y \sqrt{\gamma} \left[W^{mn}_{(\mathrm{E})mn} - P^{(A)}_{mn} P^{mn}_{(A)} \right] - 2\pi\ell^2 \chi[\Sigma]$$

Ren. Area [Alexakis-Mazzeo, 2010]/ Ren. HEE [Anastasiou-Araya-RO, 2018]

$$\mathcal{A}_{\text{ren}}[\Sigma] = \frac{\ell^2}{4} \int\limits_{\Sigma} d^2y \sqrt{\gamma} \delta^{pq}_{mn} \left(\mathcal{R}^{mn}_{pq} + \frac{1}{\ell^2} \delta^{mn}_{pq} \right) - 2\pi\ell^2 \chi[\Sigma]$$

Renormalized Area

Minimal surface/Einstein ambient space

$$\mathcal{A}_{\text{ren}} = L_{\Sigma_{\text{min}}}[E]$$

Renormalized Area

$$\mathcal{A}_{ren}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left[W^{mn}_{(E)mn} - P^{(A)}_{mn}P^{mn}_{(A)}\right] - 2\pi\ell^2\chi\left[\Sigma\right]$$

Ren. Area [Alexakis-Mazzeo, 2010], Ren. HEE [Anastasiou-Araya-Olea, 2018]

$$\mathcal{A}_{ren}\left[\Sigma\right] = \frac{\ell^2}{4}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\,\delta^{pq}_{mn}\left(\mathcal{R}^{mn}_{pq} + \frac{1}{\ell^2}\delta^{mn}_{pq}\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

Renormalized Area

Minimal surface/Einstein ambient space

$$\mathcal{A}_{\text{ren}} = L_{\Sigma_{\text{min}}}[E]$$

Renormalized Area

$$\mathcal{A}_{ren}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left[W^{mn}_{(\text{E})mn} - P^{(A)}_{mn}P^{mn}_{(A)}\right] - 2\pi\ell^2\chi\left[\Sigma\right]$$

Ren. Area [Alexakis–Mazzeo, 2010]/Ren. HEE [Anastasiou–Araya–RO, 2018]

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right] = \frac{\ell^2}{4}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\,\delta^{pq}_{mn}\left(\mathcal{R}^{mn}_{pq} + \frac{1}{\ell^2}\delta^{mn}_{pq}\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

Renormalized Area

Minimal surface/Einstein ambient space

$$\mathcal{A}_{\mathsf{ren}} = L_{\Sigma_{\mathsf{min}}}[E]$$

Renormalized Area

$$\mathcal{A}_{ren}[\Sigma] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left[W^{mn}_{(\mathsf{E})mn} - P^{(A)}_{mn}P^{mn}_{(A)}\right] - 2\pi\ell^2\chi[\Sigma]$$

Ren. Area [Alexakis-Mazzeo, 2010], Ren. HEE [Anastasiou-Araya-Olea, 2018]

$$\mathcal{A}_{ren}[\Sigma] = \frac{\ell^2}{4}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\,\delta^{pq}_{mn}\left(\mathcal{R}^{mn}_{pq} + \frac{1}{\ell^2}\delta^{mn}_{pq}\right) - 2\pi\ell^2\chi[\Sigma]$$

Renormalized Area

Minimal surface/Einstein ambient space

$$\mathcal{A}_{\text{ren}} = L_{\Sigma_{\text{min}}}[E]$$

Renormalized Area

$$\mathcal{A}_{ren}[\Sigma] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left[W^{mn}_{(\text{E})mn} - P^{(A)}_{mn}P^{mn}_{(A)}\right] - 2\pi\ell^2\chi[\Sigma]$$

Ren. Area [Alexakis-Mazzeo, 2010]/ Ren. HEE [Anastasiou-Araya-RO, 2018]

$$\mathcal{A}_{\text{ren}}[\Sigma] = \frac{\ell^2}{4}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\delta^{pq}_{mn}\left(\mathcal{R}^{mn}_{pq} + \frac{1}{\ell^2}\delta^{mn}_{pq}\right) - 2\pi\ell^2\chi[\Sigma]$$

Renormalized Area

Minimal surface/Einstein ambient space

$$\mathcal{A}_{\mathsf{ren}} = L_{\Sigma_{\mathsf{min}}}[E]$$

Renormalized Area

$$\mathcal{A}_{ren}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left[W^{mn}_{(\mathsf{E})mn} - P^{(A)}_{mn}P^{mn}_{(A)}\right] - 2\pi\ell^2\chi\left[\Sigma\right]$$

Ren. Area [Alexakis-Mazzeo, 2010]/ Ren. HEE [Anastasiou-Araya-RO, 2018]

$$\mathcal{A}_{\mathtt{ren}}\left[\Sigma\right] = \frac{\ell^2}{4}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\delta^{pq}_{mn}\left(\mathcal{R}^{mn}_{pq} + \frac{1}{\ell^2}\delta^{mn}_{pq}\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

Willmore Energy

Functional defined on compact and orientable 2D surfaces immersed in $\mathbb{R}^3$

In terms of the (spatial) mean curvature

$$\mathcal{W}[\Sigma] = \int_{\Sigma} d^2y \sqrt{\gamma} H^2$$

Willmore Energy

Functional defined on compact and orientable 2D surfaces immersed in $\mathbb{R}^3$

In terms of the (spatial) mean curvature

$$\mathcal{W}[\Sigma] = \int\limits_{\Sigma} d^2y\sqrt{\gamma}H^2$$

Willmore Energy

Functional defined on compact and orientable 2D surfaces immersed in $\mathbb{R}^3$

In terms of the (spatial) mean curvature

$$\mathcal{W}[\Sigma] = \int_{\Sigma} d^2y \sqrt{\gamma} H^2$$

Willmore Energy

Functional defined on compact and orientable 2D surfaces immersed in $\mathbb{R}^3$

In terms of the (spatial) mean curvature

$$\mathcal{W}[\Sigma] = \int_{\Sigma} d^2y \sqrt{\gamma} H^2$$

Willmore Energy from Renormalized Area

Renormalized Area

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left(W^{mn}_{(\mathrm{E})mn} - P^{(A)}_{mn}P^{mn}_{(A)}\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

For pure/global AdS$_4$ as ambient space: constant-time slice

$$W = 0 \qquad\qquad K^{(t)} = 0$$

Gauss-Codazzi relations

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right] = -\frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left(R^{ij}_{ij} - \mathcal{R} + 2H^2\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

Willmore Energy from Renormalized Area

Renormalized Area

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right]=\frac{\ell^{2}}{2}\int\limits_{\Sigma}d^{2}y\sqrt{\gamma}\left(W_{(\mathsf{E})mn}^{mn}-P_{mn}^{(A)}P_{(A)}^{mn}\right)-2\pi\ell^{2}\chi\left[\Sigma\right]$$

For pure/global AdS$_4$ as ambient space, constant-time slice

$$W=0 \qquad K^{(t)}=0$$

Gauss-Codazzi relations

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right]=-\frac{\ell^{2}}{2}\int\limits_{\Sigma}d^{2}y\sqrt{\gamma}\left(R_{ij}^{ij}-\mathcal{R}+2H^{2}\right)-2\pi\ell^{2}\chi\left[\Sigma\right]$$

Willmore Energy from Renormalized Area

Renormalized Area

$$\mathcal{A}_{\mathsf{ren}}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left(W^{mn}_{(\mathsf{E})mn} - P^{(A)}_{mn}P^{mn}_{(A)}\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

For pure/global AdS_4 as ambient space, constant-time slice

$$W = 0 \qquad \mathcal{K}^{(t)} = 0$$

Gauss-Codazzi relations

$$\mathcal{A}_{\mathsf{ren}}\left[\Sigma\right] = -\frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left(R^{ij}_{ij} - \mathcal{R} + 2H^2\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

Willmore Energy from Renormalized Area

Renormalized Area

$$\mathcal{A}_{\mathrm{ren}}\left[\Sigma\right]=\frac{\ell^{2}}{2}\int\limits_{\Sigma}d^{2}y\sqrt{\gamma}\left(W_{(\mathrm{E})mn}^{mn}-P_{mn}^{(A)}P_{(A)}^{mn}\right)-2\pi\ell^{2}\chi\left[\Sigma\right]$$

For pure/global AdS_4 as ambient space, constant-time slice

$$W=0 \qquad\qquad \mathcal{K}^{(t)}=0$$

Gauss-Codazzi relations

$$\mathcal{A}_{\mathrm{ren}}\left[\Sigma\right]=-\frac{\ell^{2}}{2}\int\limits_{\Sigma}d^{2}y\sqrt{\gamma}\left(R_{ij}^{ij}-\mathcal{R}+2H^{2}\right)-2\pi\ell^{2}\chi\left[\Sigma\right]$$

Willmore Energy from Renormalized Area

Renormalized Area

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right]=\frac{\ell^2}{2}\int\limits_{\Sigma}d^2y\sqrt{\gamma}\left(W^{mn}_{(\mathsf{E})mn}-P^{(A)}_{mn}P^{mn}_{(A)}\right)-2\pi\ell^2\chi\left[\Sigma\right]$$

For pure/global AdS_4 as ambient space, constant-time slice

$$W=0 \qquad\qquad \mathcal{K}^{(t)}=0$$

Gauss-Codazzi relations

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right]=-\frac{\ell^2}{2}\int\limits_{\Sigma}d^2y\sqrt{\gamma}\left(R^{ij}_{ij}-\mathcal{R}+2H^2\right)-2\pi\ell^2\chi\left[\Sigma\right]$$

Willmore Energy from Renormalized Area

Renormalized Area

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right]=\frac{\ell^{2}}{2}\int\limits_{\Sigma}d^{2}y\sqrt{\gamma}\left(W_{(\mathrm{E})mn}^{mn}-P_{mn}^{(A)}P_{(A)}^{mn}\right)-2\pi\ell^{2}\chi\left[\Sigma\right]$$

For pure/global AdS_4 as ambient space, constant-time slice

$$W=0 \qquad \mathcal{K}^{(t)}=0$$

Gauss-Codazzi relations

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right]=-\frac{\ell^{2}}{2}\int\limits_{\Sigma}d^{2}y\sqrt{\gamma}\left(R_{ij}^{ij}-\mathcal{R}+2H^{2}\right)-2\pi\ell^{2}\chi\left[\Sigma\right]$$

Willmore Energy

In the conformal frame $\hat{g}_{\mu\nu}$

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\hat{\gamma}}\left(\hat{\mathcal{R}} - 2\hat{H}^2\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

For a compact surface

$$\int\limits_{\Sigma_{\text{comp}}} d^2y\sqrt{\hat{\gamma}}\hat{\mathcal{R}} = 4\pi\chi\left[\Sigma_{\text{comp}}\right]$$

Willmore Energy [Anastasiou, Moreno, RO, Rivera-Betancour, 2020]

$$\mathcal{A}_{\text{ren}}\left[\Sigma_{\text{comp}}\right] = -\ell^2\mathcal{W}\left[\Sigma_{\text{comp}}\right]$$

Willmore Energy

In the conformal frame $\hat{g}_{\mu\nu}$

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\hat{\gamma}}\left(\hat{\mathcal{R}} - 2\hat{H}^2\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

For a compact surface

$$\int\limits_{\Sigma_{\text{comp}}} d^2y\sqrt{\hat{\gamma}}\hat{\mathcal{R}} = 4\pi\chi\left[\Sigma_{\text{comp}}\right]$$

Willmore Energy [Anastasiou, Moreno, RO, Rivera-Betancour, 2020]

$$\mathcal{A}_{\text{ren}}\left[\Sigma_{\text{comp}}\right] = -\ell^2 W\left[\Sigma_{\text{comp}}\right]$$

Willmore Energy

In the conformal frame $\hat{g}_{\mu\nu}$

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\hat{\gamma}}\left(\hat{\mathcal{R}} - 2\hat{H}^2\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

For a compact surface

$$\int\limits_{\Sigma_{\text{comp}}} d^2y\sqrt{\hat{\gamma}}\hat{\mathcal{R}} = 4\pi\chi\left[\Sigma_{\text{comp}}\right]$$

Willmore Energy [Anastasiou, Moreno, RO, Rivera-Betancour, 2020]

$$\mathcal{A}_{\text{ren}}\left[\Sigma_{\text{comp}}\right] = -\ell^2\mathcal{W}\left[\Sigma_{\text{comp}}\right]$$

Willmore Energy

In the conformal frame $\hat{g}_{\mu\nu}$

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\hat{\gamma}}\left(\hat{\mathcal{R}} - 2\hat{H}^2\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

For a compact surface

$$\int\limits_{\Sigma_{\text{comp}}} d^2y\sqrt{\hat{\gamma}}\hat{\mathcal{R}} = 4\pi\chi\left[\Sigma_{\text{comp}}\right]$$

Willmore Energy [Anastasiou, Moreno, RO, Rivera-Betancour, 2020]

$$\mathcal{A}_{\text{ren}}\left[\Sigma_{\text{comp}}\right] = -\ell^2\mathcal{W}\left[\Sigma_{\text{comp}}\right]$$

Willmore Energy

In the conformal frame $\hat{g}_{\mu\nu}$

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\hat{\gamma}}\left(\hat{\mathcal{R}} - 2\hat{H}^2\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

For a compact surface

$$\int\limits_{\Sigma_{\text{comp}}} d^2y\sqrt{\hat{\gamma}}\hat{\mathcal{R}} = 4\pi\chi\left[\Sigma_{\text{comp}}\right]$$

Willmore Energy [Anastasiou, Moreno, RO, Rivera-Betancour, 2020]

$$\mathcal{A}_{\text{ren}}\left[\Sigma_{\text{comp}}\right] = -\ell^2\mathcal{W}\left[\Sigma_{\text{comp}}\right]$$

Willmore Energy

In the conformal frame $\hat{g}_{\mu\nu}$

$$\mathcal{A}_{\text{ren}}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\hat{\gamma}}\left(\hat{\mathcal{R}} - 2\hat{H}^2\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

For a compact surface

$$\int\limits_{\Sigma_{\text{comp}}} d^2y\sqrt{\hat{\gamma}}\hat{\mathcal{R}} = 4\pi\chi\left[\Sigma_{\text{comp}}\right]$$

Willmore Energy [Anastasiou, Moreno, RO, Rivera-Betancour, 2020]

$$\mathcal{A}_{\text{ren}}\left[\Sigma_{\text{comp}}\right] = -\ell^2\mathcal{W}\left[\Sigma_{\text{comp}}\right]$$

Bonus Track: Reduced Hawking Mass

Arbitrary Σ, Einstein ambient space

$$L_{\Sigma}\left[E\right]=\frac{\ell^{2}}{4}I_{H}\left[\Sigma\right]-2\pi\ell^{2}\chi\left[\Sigma\right]$$

Reduced Hawking Mass I_H [Fischetti and Wiseman, 2016]

$$I_{H}\left[\Sigma\right]=2\int\limits_{\Sigma}d^{2}y\sqrt{\gamma}\left[\mathcal{R}+\frac{2}{\ell^{2}}-\frac{1}{2}\left(\mathcal{K}^{(A)}\right)^{2}\right]$$

Generalizes Renormalized Area Functional

$$L_{\Sigma}\left[E\right]=\mathcal{A}_{\text{ren}}\left[\Sigma\right]-\frac{\ell^{2}}{4}\int\limits_{\Sigma}d^{2}y\sqrt{\gamma}\left(\mathcal{K}^{(A)}\right)^{2}$$

Bonus Track: Reduced Hawking Mass

Arbitrary Σ, Einstein ambient space

$$L_{\Sigma}\left[E\right]=\frac{\ell^{2}}{4}I_{H}\left[\Sigma\right]-2\pi\ell^{2}\chi\left[\Sigma\right]$$

Reduced Hawking Mass I_H [Fischetti and Wiseman, 2016]

$$I_{H}\left[\Sigma\right]=2\int_{\Sigma}d^{2}y\sqrt{\gamma}\left[\mathcal{R}+\frac{2}{\ell^{2}}-\frac{1}{2}\left(\mathcal{K}^{(A)}\right)^{2}\right]$$

Generalizes Renormalized Area Functional

$$L_{\Sigma}\left[E\right]=\mathcal{A}_{\text{ren}}\left[\Sigma\right]-\frac{\ell^{2}}{4}\int_{\Sigma}d^{2}y\sqrt{\gamma}\left(\mathcal{K}^{(A)}\right)^{2}$$

Bonus Track: Reduced Hawking Mass

Arbitrary Σ, Einstein ambient space

$$L_{\Sigma}\left[E\right] = \frac{\ell^2}{4} I_H\left[\Sigma\right] - 2\pi\ell^2\chi\left[\Sigma\right]$$

Reduced Hawking Mass I_H [Fischetti and Wiseman, 2016]

$$I_H\left[\Sigma\right] = 2\int_{\Sigma} d^2y\sqrt{\gamma}\left[\mathcal{R} + \frac{2}{\ell^2} - \frac{1}{2}\left(\mathcal{K}^{(A)}\right)^2\right]$$

Generalizes Renormalized Area Functional

$$L_{\Sigma}\left[E\right] = \mathcal{A}_{\text{ren}}\left[\Sigma\right] - \frac{\ell^2}{4}\int_{\Sigma} d^2y\sqrt{\gamma}\left(\mathcal{K}^{(A)}\right)^2$$

Bonus Track: Reduced Hawking Mass

Arbitrary Σ, Einstein ambient space

$$L_{\Sigma}\left[E\right] = \frac{\ell^2}{4} I_H\left[\Sigma\right] - 2\pi\ell^2\chi\left[\Sigma\right]$$

Reduced Hawking Mass I_H [Fischetti and Wiseman, 2016]

$$I_H\left[\Sigma\right] = 2\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left[\mathcal{R} + \frac{2}{\ell^2} - \frac{1}{2}\left(\mathcal{K}^{(A)}\right)^2\right]$$

Generalizes Renormalized Area Functional

$$L_{\Sigma}\left[E\right] = \mathcal{A}_{\mathrm{ren}}\left[\Sigma\right] - \frac{\ell^2}{4}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left(\mathcal{K}^{(A)}\right)^2$$

Bonus Track: Reduced Hawking Mass

Arbitrary Σ, Einstein ambient space

$$L_{\Sigma}\left[E\right]=\frac{\ell^{2}}{4}I_{H}\left[\Sigma\right]-2\pi\ell^{2}\chi\left[\Sigma\right]$$

Reduced Hawking Mass I_H [Fischetti and Wiseman, 2016]

$$I_{H}\left[\Sigma\right]=2\int\limits_{\Sigma}d^{2}y\sqrt{\gamma}\left[\mathcal{R}+\frac{2}{\ell^{2}}-\frac{1}{2}\left(\mathcal{K}^{(A)}\right)^{2}\right]$$

Generalizes Renormalized Area Functional

$$L_{\Sigma}\left[E\right]=\mathcal{A}_{\text{ren}}\left[\Sigma\right]-\frac{\ell^{2}}{4}\int\limits_{\Sigma}d^{2}y\sqrt{\gamma}\left(\mathcal{K}^{(A)}\right)^{2}$$

Bonus Track: Reduced Hawking Mass

Arbitrary Σ, Einstein ambient space

$$L_{\Sigma}[E] = \frac{\ell^2}{4} I_H[\Sigma] - 2\pi\ell^2 \chi[\Sigma]$$

Reduced Hawking Mass I_H [Fischetti and Wiseman, 2016]

$$I_H[\Sigma] = 2\int_{\Sigma} d^2y\sqrt{\gamma}\left[\mathcal{R} + \frac{2}{\ell^2} - \frac{1}{2}\left(\mathcal{K}^{(A)}\right)^2\right]$$

Generalizes Renormalized Area Functional

$$L_{\Sigma}[E] = \mathcal{A}_{\text{ren}}[\Sigma] - \frac{\ell^2}{4}\int_{\Sigma} d^2y\sqrt{\gamma}\left(\mathcal{K}^{(A)}\right)^2$$

Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???)

Renormalized Volume $\Longrightarrow$ Renormalized Area (in conically singular manifolds)

Conformal Gravity in higher even dimensions $D \geq 8$? (with N. Boulanger)

Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???)

Renormalized Volume $\Longrightarrow$ Renormalized Area (in conically singular manifolds)

Conformal Gravity in higher even dimensions $D \geq 8$? (with N. Boulanger)

Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???)

Renormalized Volume $\Longrightarrow$ Renormalized Area (in conically singular manifolds)

Conformal Gravity in higher even dimensions $D > 4$? (with N. Boulanger)

Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???)

Renormalized Volume $\Longrightarrow$ Renormalized Area (in conically singular manifolds)

Conformal Gravity in higher even dimensions $D \geq 8$? (with N. Boulanger)

Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???)

Renormalized Volume $\Longrightarrow$ Renormalized Area (in conically singular manifolds)

Conformal Gravity in higher even dimensions $D = 2N$ (with N. Boulanger)

Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???)

Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???)

Renormalized Volume $\Longrightarrow$ Renormalized Area (in conically singular manifolds)

Conformal Gravity in higher even dimensions $D \geq 8$? (with N. Boulanger)

Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???)

Renormalized Volume $\Longrightarrow$ Renormalized Area (in conically singular manifolds)

Conformal Gravity in higher even dimensions $D \geq 8$? (with N. Boulanger)

Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???)

Renormalized Volume $\Longrightarrow$ Renormalized Area (in conically singular manifolds)

Conformal Gravity in higher even dimensions $D \geq 8$? (with N.Boulanger)

Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???)

Renormalized Volume $\Longrightarrow$ Renormalized Area (in conically singular manifolds)

Conformal Gravity in higher even dimensions $D \geq 8$? (with N.Boulanger)

Acknowledgements

Agencia Nacional de Investigación y Desarrollo

Ministerio de Ciencia, Tecnología, Conocimiento e Innovación

Gobierno de Chile

ANID/ACT210100 Anillo

HOLOGRAPHYCL

Anillos de Investigación en Ciencia y/o Tecnología