Higher Structures & Geometry for Gauge Systems

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Happy Birthday Maja!!!

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Why & What For?

- Quantization of gauge systems $\rightsquigarrow$ BV/BRST formalism
- Field theories with p-form fields $\rightsquigarrow$ Higher Gauge Theories
- Generalizations of Global Symmetries - Topological Operators
- Generalized geometric models for Gravity & (Quantum?) Spacetime

BV/BRST

Three general possible features for gauge systems:

- Gauge algebra closes only on-shell
- Field-dependent structure functions
- Reducibility of gauge generators

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For Yang-Mills-like models of ordinary gauge fields (e.g. SM), none of these applies ...

needless to say, still important classically (conservation laws, interactions, gauge-fix) and quantumly (anomalies, renormalization)

Initially developed for rather complicated theories.

What is the simplest theory with all these features?

Dilaton Gravity (2D)

- Models of 2D gravity (JT, R^2, ...) elegantly unify into the Poisson sigma model.
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- Almost standard gauge theory, but "non-linear"; the gauge symmetries are

$$\delta X^\mu = \Pi^{\mu\nu}(X)\epsilon_\mu \quad \text{and} \quad \delta A_\mu = \mathrm{d}\epsilon_\mu + \partial_\mu \Pi^{\nu\rho}(X) A_\nu \epsilon_\rho \, .$$

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- The gauge algebra contains functions and (generically) closes only on-shell

$$[\delta_1, \delta_2] A_\mu = \delta_{12} A_\mu + \partial_\mu \partial_\nu \Pi^{\rho\sigma}(X)\epsilon_\rho \epsilon_\sigma (\mathrm{d}X^\nu + \Pi^{\nu\kappa} A_\kappa)\,, \quad \epsilon_{12\mu} = \partial_\mu \Pi^{\nu\rho}(X)\epsilon_{1\nu}\epsilon_{2\rho}\,.$$

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- 1st-class constrained Hamiltonian system.
  Cf. 4D GR: functions in constraint algebra Blohmann, Barbosa Fernandes, Weinstein '10 nicely explained in Bojowald's book
- Not to forget: Kontsevich $\star$-product is computed by a PSM ∂ correlator Cattaneo, Felder '99

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But we forgot the third feature: reducibility ...

Climb to 3D

- 2 BF theories: scalar/2-form and 1-form/1-form (a.k.a. Chern-Simons): couple them.

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$$S_{\mathrm{CSM}} = \int -B_\mu \wedge \mathrm{d}X^\mu + \frac{1}{2}\eta_{ab}A^a \wedge \mathrm{d}A^b + \rho_a^\mu(X)B_\mu \wedge A^a + \frac{1}{3!}C_{abc}(X)A^a \wedge A^b \wedge A^c \,.$$

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- Rich shift / gauge symmetries; generic gauge algebra with all 3 features.
  - Not in the Lie algebra case, nor in the "standard Courant algebroid" ($\rho = 1$, $C = 0$).
  - The simplest genuine example of all 3 features is the lifting of the PSM to a 3D one:
    Such models, even deformed by "generalised R-flux", exist in any dimension Th Ch '21; Ikeda '21

$$S_{\text{PSM3}} = \int -B_\mu \wedge \mathrm{d}X^\mu + A_\mu \wedge \mathrm{d}A^\mu + \Pi^{\mu\nu}B_\mu \wedge A_\nu + \frac{1}{2}\partial_\rho \Pi^{\mu\nu}A^\rho \wedge A_\mu \wedge A_\nu \,.$$

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- Such models capture NC / NA structure of "nongeometric" string backgrounds.
  Mylonas, Schupp, Szabo '12; ...

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A PECULIARITY: NONLINEAR OPENNESS

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- This can happen when we include Wess-Zumino-Witten terms.
  cf. the chiral Lagrangian for Goldstones in 4D (5D WZW) or the principal chiral model in 2D (3D WZW)

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  cf. the chiral Lagrangian for Goldstones in 4D (5D WZW) or the principal chiral model in 2D (3D WZW)

  - 4-form twisted Courant sigma models in 3D Hansen, Strobl '09

$$S_{\mathrm{HCSM}} = S_{\mathrm{CSM}} + \int_B \frac{1}{4!} H_{\mu\nu\rho\sigma}(X) \mathrm{d}X^\mu \wedge \mathrm{d}X^\nu \wedge \mathrm{d}X^\rho \wedge \mathrm{d}X^\sigma \, .$$

$$[\delta_1, \delta_2] B_\mu = \delta_{12} B_\mu + (\dots)_{\mu\nu} \wedge F^\nu + (\dots \partial H)_{\mu\nu\rho} F^\nu \wedge F^\rho + (\dots)_{\mu a} G^a \, .$$

Th Ch, Ikeda, Jonke '24

  - (any+1)-form twisted R-Poisson sigma models in anyD Th Ch '21
  - earlier instances, 3-form twisted PSM in 2D / more generally: Dirac SM
    without nonlinear openness Klimcik, Strobl '01; Schaller, Kotov, Strobl '04

- Price to pay: geometric structures are "twisted" & BV/BRST is harder.

Graded Geometry & AKSZ/BV

- Main idea: tensor fields = functions on graded manifolds

  e.g. shifted tangent bundle $T[1]\Sigma$: A "Q-manifold" (HVF: $|Q| = 1$ and $Q^2 = 0$)

  Coordinates: σ^m and θ^m with $\theta^m\theta^n = -\theta^n\theta^m$. Functions $\sim$ p-forms / $Q \equiv \mathrm{d} = \theta^m\partial_m$

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- If also a compatible (graded) symplectic form s.t. $L_Q\Omega = 0$ $\rightsquigarrow$ QP manifold.

- Classical theory: degree-preserving maps $\Phi : T[1]\Sigma \to \mathcal{M}$ with $\mathcal{M}$ a QP target.

- Hamiltonian $\Theta \in C^\infty(\mathcal{M})$ of degree $n+1$ with $Q = \{\Theta, \cdot\}$. $Q^2 = 0 \Rightarrow \{\Theta, \Theta\} = 0$.

- Action functional of supermaps satisfying the Classical Master Equation

$$S_{\mathrm{BV}}[\Phi] = \int_\Sigma \left(\frac{1}{2}\Omega_{ab}\Phi^a \wedge \mathrm{d}\Phi^b + \Phi^*(\Theta)\right) \quad \Rightarrow \quad (S, S)_{BV} = 0\,.$$

Examples vs. Non Examples

- $n = 1$: $\mathcal{M} = T^*[1]M \rightsquigarrow$ Poisson sigma model (scalars / 1-forms)
- $n = 2$: $\mathcal{M} \subset T^*[2]E[1] \rightsquigarrow$ Courant sigma model (scalars / 1-forms / 2-forms)
- $n = 3$: 3-brane sigma models (scalars / 1,2,3-forms)
  cf. Plebanski formulation of GR (with constraint)
- $n = n$: Ševera's Σ_n-manifolds of which (untwisted) R-Poisson "brane mechanics" are a specific slice

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- Q manifolds are not always QP ... Vanilla AKSZ does not apply. E.g. WZW terms
  also in a different direction, higher gauge theories as sigma models Grützmann, Strobl '14
- Non topological models
  but AKSZ-like: "presymplectic AKSZ" of GR4 Grigoriev, Kotov '20 "BFV/AKSZ" of EC4 Canepa, Cattaneo, Schiavina '21

Non Examples as Examples / WZW Case

When $H_{n+2} \neq 0$, geometry comes to the rescue. In physics terms: Th Ch, Ikeda, Jonke '24

- Expand in antifields, as in the traditional BV approach
- Determine the geometric meaning of the coefficients in the interactions
- "Twist" by H_{n+2} the respective geometric structures

This is facilitated by an auxiliary affine connection ∇ on a suitable algebroid.
cf. Baulieu, Losev, Nekrasov '01; Cattaneo, Felder, Tomassini '00; Ikeda, Strobl '19

Specific torsion and ("basic") curvature tensors control all the interaction coefficients
higher tensors for higher brackets in more than 3D ... Th Ch, Kodžoman, Škoda '24

DETOUR: L_∞ ALGEBROIDS

The mathematical explanation of the Q vs. QP story and the WZW terms goes as:

- Recall the “derived” bracket of vector fields on the exterior algebra of forms: Cartan

$$\iota_{[X,Y]} = [L_X, \iota_Y] = [[\mathrm{d}, \iota_X], \iota_Y]\,.$$

- Think “$\mathrm{d} = Q$” and take any homological vector field. Two constructions possible:
  Kosmann-Schwarzbach; Voronov; Roytenberg; Sheng, Zhu; Bonavolonta, Poncin; ...
  - Construct an $L_\infty[1]$ Algebroid (for $\mathcal{M}$ split) with graded symmetric brackets
    using the arity $k-1$ component of the HVF and graded vector field X_i

$$\ell_k(X_1, \ldots, X_k) = [\ldots[[{}^{k-1}Q, X_1], X_2], \ldots]\,.$$

  - Construct a dgLA (for $\mathcal{M}$ symplectic) using $\{\Theta, -\}$ and a Leibniz bracket, e.g.

$$\{f,g\}_{\text{P.B.}} = \{\{\Theta, f\}, g\} \quad \text{(Poisson)} \quad e \circ e' = \{\{\Theta, e\}, e'\} \quad \text{(Dorfman)} \quad \&\text{c.}$$

- When WZW terms are there, the 1st construction works better (with connection).

DETOUR: L_∞ ALGEBROIDS

- This $L_\infty[1]$ algebroid construction gives natural geometrical tensors.
  for QP2, cf. Gualtieri torsion, Riemann curvature, basic curvature Boffo, Schupp '19; Jurco, Vysoky '16, '23; Th Ch, Jonke '22
- Higher gauge theories as generalised σ-models based on Q (split) targets $\mathcal{M}$.
  as in Grutzmann, Strobl '14 and non Abelian gerbe examples of Ho, Matsuo '12 (in different formulation) and Strobl '16
- Not the full story, possible to have non C^∞-linear higher brackets & higher anchors
  as in homotopy Poisson/P_∞ of Voronov, see also Herbig, Herber, Seaton '21; examples in Th Ch, Kodžoman, Škoda '24

Bidifferential Bigraded Manifolds

Mixed symmetry tensor fields as functions

For bipartite tensors of degree $|\omega| = (p, q)$, consider functions on $T[1]\Sigma \oplus T[1]\Sigma$,

$$\omega_{p,q} = \frac{1}{p!q!}\,\omega_{\mu_1\ldots\mu_p\nu_1\ldots\nu_q}(x)\,\theta^{\mu_1}\ldots\theta^{\mu_p}\chi^{\nu_1}\ldots\chi^{\nu_q}\ .$$

Two separate sets of odd coordinates θ^μ and χ^μ that mutually commute by convention,

$$\theta^\mu\theta^\nu = -\theta^\nu\theta^\mu\ , \quad \chi^\mu\chi^\nu = -\chi^\nu\chi^\mu\ , \quad \theta^\mu\chi^\nu = \chi^\nu\theta^\mu\ .$$

The components of the tensor field have manifest mixed index symmetry

$$\omega_{\mu_1\ldots\mu_p\nu_1\ldots\nu_q} = \omega_{[\mu_1\ldots\mu_p][\nu_1\ldots\nu_q]}\ .$$

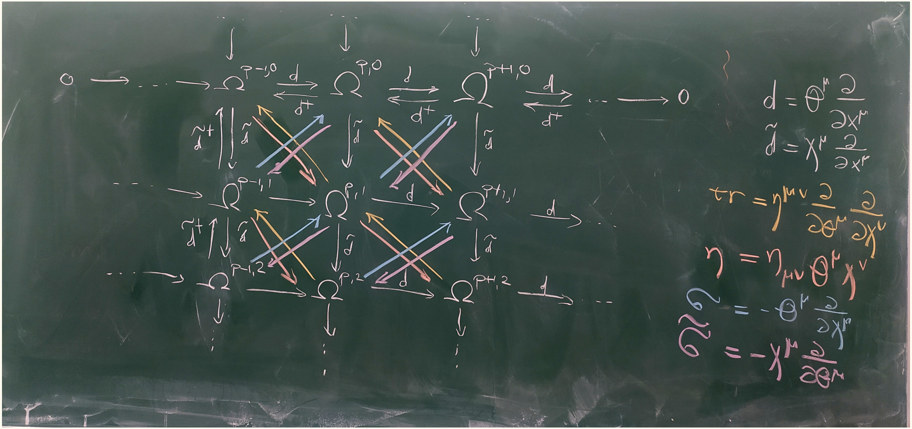
N.B. Useful to think of differential forms as bipartite tensors with 1 empty slot (p or q).

generalises to N-partite tensors; cf. the more general Ševera's differential gorms and worms, also with degree (1,1) coordinates

Two commuting homological vector fields of degree (1,0) and (0,1):

$$\mathrm{d} = \theta^\mu\frac{\partial}{\partial x^\mu} \quad \text{and} \quad \widetilde{\mathrm{d}} = \chi^\mu\frac{\partial}{\partial x^\mu} \quad \text{with} \quad \mathrm{d}^2 = 0 = \widetilde{\mathrm{d}}^2 \quad \text{and} \quad \mathrm{d}\,\widetilde{\mathrm{d}} = \widetilde{\mathrm{d}}\,\mathrm{d}\,.$$

A DOUBLE COMPLEX



graded analogon of closely related formalism by Bekaert, Boulanger, de Medeiros, Hull ... Identities as commutative diagrams

What For

- A simple & universal graded formulation of mixed symmetry tensor field theories
  - Unified formalism for scalars, *p*-forms, gravitons, the Curtright field (2,1), &c.
  - $\{$kinetic, θ, mass$\}$ terms, healthy higher-∂ interactions, nonlinear *p*-form ED

work with Giorgos Karagiannis, Fech Scen Khoo, Diederik Roest, Peter Schupp ’16-’20

- A systematic treatment of various dualities, off-shell, single and multi field
  - Universal parent action and higher duality (“Buscher”) rules
  - Generalised global symmetries as (jet) isometries & tracking of ’t Hooft anomalies
  - Off-shell duality for the graviton with θ term
  - “Axion gravitodynamics”

work with Giorgos Karagiannis, George Manolakos,Arash Ranjbar, Peter Schupp ’19-’22

Lagrangians

Generalized Hodge Duality

To construct Lagrangians, we need a suitable inner product. Generalized Hodge star:

$$(\star\,\omega)_{D-p,D-q} = \frac{1}{(D-p-q)!}\,\eta^{D-p-q}\,\omega^{\mathrm{T}}_{q,p}\,.$$

The combination $*\widetilde{*}$ (of standard Hodge) is different than $\star$ (it also encodes traces) :

$$\star\,\omega = *\,\widetilde{*}\,(-1)^{\epsilon}\sum_{n=0}^{\min(p,q)}\frac{(-1)^n}{(n!)^2}\,\eta^n\,\mathrm{tr}^n\,\omega\;, \qquad \big(\epsilon = (D-1)(p+q)+pq+1\big)\,.$$

A symmetric inner product of some ω and ω' is then simply defined by $\int_{\theta,\chi}\omega\star\omega'$.

Kinetic and Mass terms

$$\mathcal{L}_{\text{kin}}(\omega_{p,q}) = \int_{\theta,\chi} \mathrm{d}\omega \star \mathrm{d}\omega\,. \qquad \mathcal{L}_{\text{mass}}(\omega_{p,q}) = m^2 \int_{\theta,\chi} \omega \star \omega\,.$$

- For differential forms ($q = 0$) $\rightsquigarrow$ p-form electrodynamics.
- For $p = q = 1$ $\rightsquigarrow$ linearized Einstein-Hilbert / Fierz-Pauli:

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} h^{\mu}{}_{\mu} \Box h^{\nu}_{\nu} + \frac{1}{2} h^{\lambda}{}_{\lambda} \partial_\mu \partial_\nu h^{\mu\nu} - \frac{1}{2} h_{\mu\nu} \partial^\nu \partial_\lambda h^{\mu\lambda} + \frac{1}{4} h_{\mu\nu} \Box h^{\mu\nu}\,,$$

$$\mathcal{L}_{\text{mass}} = m^2 \left(h^{\mu\nu} h_{\mu\nu} - (h^{\mu}{}_{\mu})^2 \right)\,.$$

- For $p = 2, q = 1$, the gauge theory for the hook Young tableaux Curtright '80

$$\begin{aligned}
\mathcal{L}_{\text{kin}} &= \frac{1}{2} \Big(\partial_\mu \omega_{\nu\kappa|\lambda} \partial^\mu \omega^{\nu\kappa|\lambda} - 2\partial_\mu \omega^{\mu\nu|\kappa} \partial^\lambda \omega_{\lambda\nu|\kappa} - \partial_\mu \omega^{\nu\kappa|\mu} \partial^\lambda \omega_{\nu\kappa|\lambda} - \\
&\quad - 4\omega_\mu{}^{\nu|\mu} \partial^\kappa \partial^\lambda \omega_{\kappa\nu|\lambda} - 2\partial_\mu \omega_\nu{}^{\kappa|\nu} \partial^\mu \omega^\lambda{}_{\kappa|\lambda} + 2\partial_\mu \omega_\nu{}^{\mu|\nu} \partial^\kappa \omega^\lambda{}_{\kappa|\lambda} \Big)\,, \\
\mathcal{L}_{\text{mass}} &= m^2 \left(\omega^{\mu\nu|\rho} \omega_{\mu\nu|\rho} - 2\omega^\mu \omega_\mu \right)\,.
\end{aligned}$$

cf. Bergshoeff, Fernandez-Melgarejo, Rosseel, Townsend '12

Tensor Galileons as ``generalised kinetic terms''

Define $\omega_{(n+1)} \equiv \omega \left(\mathrm{d}\tilde{\mathrm{d}}\, \omega \right)^n$. For any bipartite tensor in any D, universal interactions:

$$\mathcal{L}_{\mathrm{Gal}}(\omega_{p,q}) = \sum_{n=0}^{n_{max}} \int_{\theta,\chi} \mathrm{d}\omega_{(n+1)} \star \mathrm{d}\omega_{(n+1)} \, ,$$

Single-field, 2nd-order (polynomial) EOMs. Note: only even field appearances here.

When $p = q$ (scalars, gravitons, $(2,2)$s &c.), an enhancement to odd fields

$$\widetilde{\mathcal{L}}_{\mathrm{Gal}}(\omega_{p,p}) = \mathcal{L}_{\mathrm{Gal}}(\omega) + \sum_n \int_{\theta,\chi} \eta^{p+1} \mathrm{d}\omega_{(n)} \star \mathrm{d}\omega_{(n+1)} = \sum_{n=1}^{n_{\max}} \int_{\theta,\chi} \eta^{D-(p+1)n-p}\, \omega_{(n+1)} \, .$$

straightforward to do multi-field, up-to-2nd-order EOM / also to find expanded versions (but they will be very complicated)

cf. Nicolis, Ratazzi, Trincherini '08; Deffayet, Deser, Esposito-Farese '09-'10; ...

also for higher spins, with suitable generalised Hodge; yields the formulation of Francia, Sagnotti '02 and nonlocal Galileons

Generalised Global Symmetries

Global Symmetries & 't Hooft Amomaly

Gaiotto, Kapustin, Seiberg, Willet '14; Cordova, Dumitrescu, Intriligator '18

- Free Maxwell theory has two 1-form $U(1)$ global symmetries (electric/magnetic)
  - The conserved currents are the 2-forms: F and $*F$ give topological operators, act on Wilson/'t Hooft
  - Background fields that can couple to the currents are two 2-forms B_e and B_m
  - They have background gauge transformation: $B_\mathrm{e/m} \to B_\mathrm{e/m} + \mathrm{d}\Lambda_\mathrm{e/m}$
  - Electric description: $U(1)$ gauge field A, which shifts by Λ_e but is inert under magnetic

- Action coupled to background fields we can add the ϑ term too

$$S = \frac{1}{2e^2}\int (F - B_\mathrm{e}) \wedge *(F - B_\mathrm{e}) + \frac{i}{2\pi}\int B_\mathrm{m} \wedge F\,.$$

  - Under background gauge transformations: $S \to S + \frac{i}{2\pi}\int \Lambda_\mathrm{e} \wedge \mathrm{d}B_\mathrm{m}$.
  - Mixed $U(1)_\mathrm{e}$-$U(1)_\mathrm{m}$ 't Hooft anomaly via inflow from 6D polynomial $\mathrm{d}B_\mathrm{m} \wedge \mathrm{d}B_\mathrm{e}$
  - In the dual theory the currents are exchanged, the anomaly is reproduced

- Anomaly matching also for scalars, multiple fields, nonlinear theories ...

  Th Ch, Karagiannis, Ranjbar '22

Graded Isometries

- Philosophy of Generalised Global Symmetries: the photon is a NG boson.
- Nonlinear sigma models were introduced for (scalar) NG bosons. Gell-Mann, Levy '60
- Q-manifold Philosophy: Generalised sigma models with graded coordinates.
- In 2D NLSMs global symmetries are identified by background isometries:

$$\delta X = \rho(\epsilon) \quad \text{iff} \quad \mathcal{L}_\rho G = 0 \quad \text{and} \quad \mathcal{L}_\rho B = \mathrm{d}\alpha\,.$$

- Similarly when the couplings depend on derivatives too (see Heisenberg pion fireball model):

$$\delta X = \rho(\epsilon) \quad \text{and} \quad \delta \mathrm{d}X = \xi(\epsilon) \quad \text{iff} \quad \widehat{\mathcal{L}}_V G = 0 \quad \text{and} \quad \widehat{\mathcal{L}}_V B = \mathrm{d}\beta\,,$$

  with V in the 1-jet and $\widehat{\mathcal{L}}$ the 1-jet Lie derivative.
- For Abelian 1-forms in 4D, same-yet-graded result! a graded Lie derivative along a graded VF

OPEN QUESTIONS

- Is the graviton a Nambu-Goldstone boson for some global symmetry?
  for a proposal on this see Hinterbichler, Hofman, Joyce, Mathys '23
- Does linGR have a 't Hooft anomaly? How does it inflow?
- What are the topological operators and the extended observables then?
  tensor gauge theories relevant elsewhere too, e.g. physics of reduced mobility quasiparticles (fractons, lineons, planons)
- Is there a Coleman-Mermin-Wagner theorem for gravitons?
- Do mixed symmetry tensors gauge theories arise as bg fields for such GGSs?
- ...

THANK YOU

&

HAPPY BIRTHDAY MAJA!!!