# A realistic and testable Supersymmetric Model from the Dimensional Reduction of an $N = 1$, $10D$, $E_8$ Theory over a Modified Flag Manifold

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Quantum & fuzzy:
Workshop in honour of the 65th birthday of Professor Maja Burić

4 April 2024, Belgrade, Serbia

1. Short reminder of the Kaluza - Klein programme
2. Higher-Dimensional Unified Gauge Theories and Coset Space Dimensional Reduction (CSDR)
3. A realistic Effective Theory
4. Embedding in the heterotic $10D$ Superstring

1. Fuzzy extra dimensions → renormalizable realistic 4-d GUTs
2. Reduction of couplings in $\mathcal{N} = 1$ gauge theories → predictive GUTs, Finite Unified Theories, reduced MSSM
3. Noncommutative (fuzzy) Gravity
4. Ghost-free conformal Gravity, $R^2$ Gravity
5. Unification of Gravity theories with internal interactions


ZAROS

# Kaluza - Klein

- Kaluza-Klein observation: Dimensional Reduction of a pure gravity theory on $M^4 \times S^1$ leads to a $U(1)$ gauge theory coupled to gravity in four dimensions. The extra dimensional gravity provided a geometrical unified picture of gravitation and electromagnetism.
- Generalization to $M^D = M^4 \times B$, with $B$ a compact Riemannian space with a non-abelian isometry group $S$ leads after dim. reduction to gravity coupled to Y-M in 4 dims.

*Kerner* '68

*Cho - Freund* '75

# Problems

- No classical ground state corresponding to the assumed $M^D$.

- Adding fermions in the original action, it is impossible to obtain chiral fermions in four dims.

  *Witten* '85

- However by adding suitable matter fields in the original action, in particular Y-M one can have a classical stable ground state of the required form and massless chiral fermions in four dims.

  *Horvath - Palla - Cremmer - Scherk* '77

## Original motivation

Use higher dimensions

- to unify the gauge and Higgs sectors
- to unify the fermion interactions with gauge and Higgs fields

⋆ Supersymmetry provides further unification (fermions in adj. reps)

*Forgacs - Manton* '79, *Manton* '81, *Chapline - Slansky* '82
*Kubyshin - Mourao - Rudolph - Volobujev* '89
*Kapetanakis - Z* '92, *Manousselis - Z* '01 – '08

## Further successes

($a$) chiral fermions in $4$ dims from vector-like reps in the higher dim theory

($b$) the metric can be deformed (in certain non-symmetric coset spaces) and more than one scales can be introduced

($c$) Wilson flux breaking can be used

($d$) Softly broken susy chiral theories in $4$ dims can result from a higher dimensional susy theory

Theory in $D$ dims $\rightarrow$ Theory in 4 dims

1. Compactification

$$\begin{array}{ccccc} M^D & \rightarrow & M^4 & \times & B \\ | & & | & & | \\ x^M & & x^\mu & & y^a \end{array}$$

$B$ - a compact space
$\dim B = D - 4 = d$

2. Dimensional Reduction

Demand that $\mathcal{L}$ is independent of the extra $y^a$ coordinates

- One way: Discard the field dependence on $y^a$ coordinates
- An elegant way: Allow field dependence on $y^a$ and employ a symmetry of the Lagrangian to compensate

  Obvious choice: Gauge Symmetry

Allow a non-trivial dependence on $y^a$, but impose the condition that a symmetry transformation by an element of the isometry group $S$ of $B$ is compensated by a gauge transformation.

$\Rightarrow \mathcal{L}$ independent of $y^a$ just because is gauge invariant.

Integrate out extra coordinates

CSDR: $B = S/R$

$$S: \quad Q_A = \{\underbrace{Q_i}_{R}, \underbrace{Q_a}_{S/R}\}$$

$$[Q_i, Q_j] = f_{ij}^k Q_k\,, [Q_i, Q_a] = f_{ia}^b Q_b\,,$$
$$[Q_a, Q_b] = f_{ab}^i Q_i + f_{ab}^c Q_c\,,$$

where $f_{ab}^c$ vanishes in symmetric $S/R$

Consider a Yang-Mills-Dirac theory in $D$ dims based on group $G$ defined on $M^D \to M^4 \times S/R$, $D = 4 + d$

$$g^{MN} = \begin{pmatrix} \eta^{\mu\nu} & 0 \\ 0 & -g^{ab} \end{pmatrix} \qquad \eta^{\mu\nu} = diag(1,-1,-1,-1)$$

$$d = dimS - dimR \qquad g^{ab} - \text{coset space metric}$$

$$A = \int \mathrm{d}^4x\mathrm{d}^dy\sqrt{-g}\left[-\frac{1}{4}Tr(F_{MN}F_{K\Lambda})g^{MK}g^{N\Lambda} + \frac{i}{2}\overline{\psi}\Gamma^M D_M\psi\right]$$

$$D_M = \partial_M - \theta_M - A_M \quad , \quad \theta_M = \frac{1}{2}\theta_{MN\Lambda}\Sigma^{N\Lambda}$$

where $\theta_M$ is the spin connection of $M^D$ and $\psi$ is in rep $F$ of $G$

We require that any transformation by an element of $S$ acting on $S/R$ is compensated by gauge transformations.

$$
\begin{aligned}
A_\mu(x,y) =& g(s) A_\mu(x, s^{-1}y) g^{-1}(s) \\
A_a(x,y) =& g(s) J_a{}^b A_b(x, s^{-1}y) g^{-1}(s) \\
& + g(s) \partial_a g^{-1}(s) \\
\psi(x,y) =& f(s) \Omega \psi(x, s^{-1}y) f^{-1}(s)
\end{aligned}
$$

$g, f$ - gauge transformations in the adj, $F$ of $G$ corresponding to the $s$ transformation of $S$ acting on $S/R$

$J_a{}^b$ - Jacobian for $s$

$\Omega$ - Jacobian + local Lorentz rotation in tangent space

Above conditions imply constraints that $D$-dims fields should obey.

Solution of constraints:

- 4-dim fields
- Potential
- Remaining gauge invariance

Taking into account all the constraints and integrating out the extra coordinates, we obtain in 4 dims:

$$A = C\int \mathrm{d}^4x \left(-\frac{1}{4}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\sum_a \mathrm{Tr}(D_\mu\phi_a D^\mu\phi^a)\right.$$
$$\left. + V(\phi) + \frac{i}{2}\bar{\psi}\Gamma^\mu D_\mu\psi - \frac{i}{2}\bar{\psi}\Gamma^a D_a\psi\right)$$

kinetic terms        mass terms

$$D_\mu = \partial_\mu - A_\mu\,,\ D_a = \partial_a - \theta_a - \phi_a\,,\ \theta_a = \frac{1}{2}\theta_{abc}\Sigma^{bc}$$

$C-$ volume of cs , $\theta_a-$ spin connection of cs

$$V(\phi) = -\frac{1}{4}g^{ac}g^{bd}\mathrm{Tr}\left\{\left(f_{ab}^C\phi_C - [\phi_a,\phi_b]\right)\left(f_{cd}^D\phi_D - [\phi_c,\phi_d]\right)\right\}$$

$A = 1,\ldots,\mathrm{dim}S$, $f-$ structure constants of $S$.
Still $V(\phi)$ only formal since $\phi_a$ must satisfy $f_{ai}^D\phi_D - [\phi_a,\phi_i] = 0$.

1) The 4-dim gauge group

$$H = C_G(R_G)$$
$$\text{i.e.} \quad G \supset R_G \times H$$

where $G$ is the higher-dim group and $H$ is the 4 dim group.

2) Scalar fields

$$S \supset R$$
$$\text{adj}S = \text{adj}R + \upsilon$$
$$G \supset R_G \times H$$
$$\text{adj}G \supset (\text{adj}R, 1) + (1, \text{adj}H) + \Sigma(r_i, h_i)$$

If $\upsilon = \Sigma s_i$

when $s_i = r_i \Rightarrow$ $h_i$ survives in 4 dims.

## 3) Fermions

$$G \supset R_G \times H$$
$$F = \sum (t_i, h_i)$$

spinor of $SO(d)$ under $R$

$$\sigma_d = \sum \sigma_i$$

for every $t_i = \sigma_i \Rightarrow$ $h_i$ survives in 4 dims.

Possible to obtain a chiral theory in 4 dims starting from Weyl fermions in a complex rep.
However, even starting with Weyl (+ Majorana) fermions in vector-like reps of $G$ in $D = 4n + 2$ dims we are also led to a chiral theory.

If $D$ is even:

$$\Gamma^{D+1}\Psi_\pm = \pm\Psi_\pm \qquad \text{Weyl condition}$$
$$\Psi = \Psi_+ \oplus \Psi_- = \sigma_D + \sigma'_D\,,$$

where $\sigma_D, \sigma'_D$ are non-self conjugate spinors of $SO(1, D-1)$.

The $(SU(2)\times SU(2))\times SO(d)$ branching rule is:

$$\sigma_D = (2,1;\sigma_d) + (1,2;\sigma'_d)$$
$$\sigma'_D = (2,1;\sigma'_d) + (1,2;\sigma_d)$$

Starting with Dirac fermions

$\rightsquigarrow$ equal number of left and right-handed reps of the 4-dim group $H$

Weyl condition selects either $\sigma_D$ or $\sigma'_D$

Weyl condition cannot be applied in odd dims. In that case:

$$\sigma_D = (2, 1; \sigma_d) + (1, 2; \sigma_d)\,,$$

where $\sigma_d$ is the unique spinor of $SO(d)$

$\rightsquigarrow$ equal number of left and right-handed reps in 4 dims

Most interesting case is when $D = 4n + 2$ and we start with a vectorlike rep. In that case $\sigma_d$ is non-self-conjugate and $\sigma'_d = \bar{\sigma}_d$.

Then the decomposition of $\sigma_d, \bar{\sigma}_d$ of $SO(d)$ under $R$ is:

$$\sigma_d = \sum \sigma_k\,, \quad \bar{\sigma}_d = \sum \bar{\sigma}_k\,.$$

Then:

$$G \supset R_G \times H$$

vectorlike $\leftarrow F = \sum_i (r_i, h_i) \rightarrow$ either self-conjugate or

have a partner $(\bar{r}_i, \bar{h}_i)$.

Then according to the rule from $\sigma_d$ we will obtain in 4 dims left-handed fermions $f_L = \sum h_k^L$.

Since $\sigma_d$ is non-self-conjugate, $f_L$ is non-self-conjugate.

Similarly, from $\bar{\sigma}_d$, we obtain the right-handed rep $\sum \bar{h}_k^R = \sum h_k^L$.

Moreover since $F$ vectorlike, $\bar{h}_k^R \sim h_k^L$, i.e. $H$ is chiral theory with double spectrum.

We can still impose Majorana condition (Weyl and Majorana are compatible in $4n+2$ dims) to eliminate the doubling of the fermion spectrum.

Majorana condition (reverses the sign of all int. qu. nos) forces $f_R$ to be the charge conjugate of $f_L$.

If $F$ complex $\rightarrow$ chiral theory just $\bar{h}_k^R$ is different from $h_k^L$.

An easy case in calculating the potential, its minimization and SSB:

If $G \supset S \Rightarrow H$ breaks to $K = C_G(S)$:

$$\begin{array}{ccccl} G \supset & S & \times & K & \leftarrow \text{gauge group after SSB} \\ & \cup & & \cap & \\ G \supset & R & \times & H & \leftarrow \text{gauge group in 4 dims} \end{array}$$

But

fermion masses

$$M^2\Psi = D_a D^a \Psi - \frac{1}{4} R\Psi - \frac{1}{2} \underbrace{\Sigma^{ab} F_{ab}}_{=0 \text{, if } S \subset G} \Psi > 0$$

$$= (C_s + C_R)\Psi$$

comparable to the compactification scale.

# Supersymmetry breaking by dim reduction over symmetric CS (e.g $SO(7)/SO(6)$)

Consider $G = E_8$ in 10 dims with Weyl-Majorana fermions in the adjoint rep of $E_8$, i.e. a susy $E_8$.
Embedding of $R = SO(6)$ in $E_8$ is suggested by the decomposition:

$$
\begin{aligned}
E_8 &\supset SO(6) \times SO(10) \\
248 &= (15, 1) + (1, 45) + (6, 10) + (4, 16) + (\overline{4}, \overline{16})
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{adj}S &= \mathrm{adj}R + v \\
21 &= 15 + 6 \leftarrow \text{vector}
\end{aligned}
$$

Spinor of $SO(6)$: $4$
In 4 dims we obtain a gauge theory based on:

$$
H = C_{E_8}(SO(6)) = SO(10)\,,
$$

with scalars in $10$ and fermions in $16$.

- *Theorem*: When $S/R$ symmetric, the potential necessarily leads to spontaneous breakdown of $H$.

•• Moreover in this case we have:

$$\begin{array}{ccccc} E_8 & \supset & SO(7) & \times & SO(9) \\ & & \cup & & \cap \\ E_8 & \supset & SO(6) & \times & SO(10) \end{array}$$

$\Rightarrow$ Final gauge group after breaking:

$$K = C_{E_8}(SO(7)) = SO(9)$$

CSDR over symmetric coset spaces breaks completely original supersymmetry.

# Soft Supersymmetry Breaking by CSDR over non-symmetric CS.

We have examined the dim reduction of a supersymmetric $E_8$ over the $3$ existing $6-$dim CS:

$$G_2/SU(3)\,, \quad Sp(4)/(SU(2)\times U(1))_{\text{non-max}}\,, \quad SU(3)/U(1)\times U(1)$$

$\Rightarrow$ Softly Broken Supersymmetric Theories in 4 dims without any further assumption

Non-symmetric CS admit torsion and the two latter more than one radii.

Consider supersymmetric $E_8$ in 10 dims and $S/R = G_2/SU(3)$.

We use the decomposition:

$$E_8 \supset SU(3) \times E_6$$
$$248 = (8,1) + (1,78) + (3,27) + (\overline{3},\overline{27})$$

and choose $R = SU(3)$

$$\mathrm{adj}S = \mathrm{adj}R + v$$
$$14 = 8 + \underbrace{3 + \bar{3}}_{\text{vector}}$$

Spinor: $1 + 3$ under $R = SU(3)$

$\Rightarrow$ In 4 dim theory: $H = C_{E_8}(SU(3)) = E_6$ with:
scalars in $27 = \beta$ and fermions in $27, 78$

i.e.: spectrum of a supersymmetric $E_6$ theory in 4 dims.

The Higgs potential of the genuine Higgs $\beta$:

$$V(\beta) = 8 - \frac{40}{3}\beta^2 - [4d_{ijk}\beta^i\beta^j\beta^k + h.c.] + \beta^i\beta^j d_{ijk}d^{k\ell m}\beta_\ell\beta_m + \frac{11}{4}\sum_\alpha \beta^i(G^\alpha)_i^j\beta_j\beta^k(G^\alpha)_k^\ell\beta_\ell$$

which obtains F-terms contributions from the superpotential:

$$W(B) = \frac{1}{3}d_{ijk}B^iB^jB^k$$

D-term contributions:

$$\frac{1}{2}D^\alpha D^\alpha\,, \quad D^\alpha = \sqrt{\frac{11}{2}}\beta^i(G^\alpha)_i^j\beta_j$$

The rest terms belong to the SSB part of the Lagrangian:

$$\mathcal{L}_{scalar}^{SSB} = -\frac{1}{R^2}\frac{40}{3}\beta^2 - [4d_{ijk}\beta^i\beta^j\beta^k + h.c.]\frac{g}{R}$$

$$M_{gaugino} = (1+3\tau)\frac{6}{\sqrt{3}}\frac{1}{R}$$

# Reduction of 10-dim, $\mathcal{N}=1$, $E_8$ over $S/R = SU(3)/U(1) \times U(1) \times Z_3$

*Irges - Z '11*

*Manolakos - Patellis - Z '20*

*Patellis - Porod - Z '23*

We use the decomposition:

$$E_8 \supset E_6 \times SU(3) \supset E_6 \times U(1)_A \times U(1)_B$$

and choose $R = U(1)_A \times U(1)_B$,

$$\rightsquigarrow H = C_{E_8}(U(1)_A \times U(1)_B) = E_6 \times U(1)_A \times U(1)_B$$

$$E_8 \supset E_6 \times U(1)_A \times U(1)_B$$

$$\begin{aligned}
248 = &\ 1_{(0,0)} + 1_{(0,0)} + 1_{(3,1/2)} + 1_{(-3,1/2)} \\
&\ 1_{(0,-1)} + 1_{(0,1)} + 1_{(-3,-1/2)} + 1_{(3,-1/2)} \\
&\ 78_{(0,0)} + 27_{(3,1/2)} + 27_{(-3,1/2)} + 27_{(0,-1)} \\
&\ \overline{27}_{(-3,-1/2)} + \overline{27}_{(3,-1/2)} + \overline{27}_{(0,1)}
\end{aligned}$$

$$\text{adj}S = \text{adj}R + v \quad \leftarrow \text{vector}$$

$$\Downarrow$$

$$8 = (0,0) + (0,0) + (3, 1/2) + (-3, 1/2)$$
$$+ (0, -1) + (0, 1) + (-3, -1/2) + (3, -1/2)$$

$$SO(6) \supset SU(3) \supset U(1)_A \times U(1)_B$$
$$4 = 1 + 3 = (0,0) + (3, 1/2) + (-3, 1/2) + (0, -1)$$

spinor

# 4-dim theory

$\mathcal{N}=1, E_6\times U(1)_A\times U(1)_B$ with chiral supermultiplets:

$A^i : 27_{(3,1/2)}\ ,\ B^i : 27_{(-3,1/2)}\ ,\ C^i : 27_{(0,-1)}\ ,\ A : 1_{(3,1/2)}\ ,\ B : 1_{(-3,1/2)}\ ,\ C : 1_{(0,-1)}$

Scalar potential:

$$
\begin{aligned}
\frac{2}{g^2}V = &\frac{2}{5}\left(\frac{1}{R_1^4}+\frac{1}{R_2^4}+\frac{1}{R_3^4}\right)+\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\alpha^i\alpha_i+\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\bar{\alpha}\alpha\\
&+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\beta^i\beta_i+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\bar{\beta}\beta+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\gamma^i\gamma_i+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\bar{\gamma}\gamma\\
&+\sqrt{2}80\left[\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)d_{ijk}\alpha^i\beta^j\gamma^k+\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)\alpha\beta\gamma+h.c\right]\\
&+\frac{1}{6}\left(\alpha^i(G^\alpha)_i^j\alpha_j+\beta^i(G^\alpha)_i^j\beta_j+\gamma^i(G^\alpha)_i^j\gamma_j\right)^2\\
&+\frac{10}{6}\left(\alpha^i(3\delta_i^j)\alpha_j+\bar{\alpha}(3)\alpha+\beta^i(-3\delta_i^j)\beta_j+\bar{\beta}(-3)\beta\right)^2\\
&+\frac{40}{6}\left(\alpha^i(\tfrac{1}{2}\delta_i^j)\alpha_j+\bar{\alpha}(\tfrac{1}{2})\alpha+\beta^i(\tfrac{1}{2}\delta_i^j)\beta_j+\bar{\beta}(\tfrac{1}{2})\beta+\gamma^i(-1\delta_i^j)\gamma^j+\bar{\gamma}(-1)\gamma\right)^2\\
&+40\alpha^i\beta^jd_{ijk}d^{klm}\alpha_l\beta_m+40\beta^i\gamma^jd_{ijk}d^{klm}\beta_l\gamma_m+40\alpha^i\gamma^jd_{ijk}d^{klm}\alpha_l\gamma_m\\
&+40(\bar{\alpha}\bar{\beta})(\alpha\beta)+40(\bar{\beta}\bar{\gamma})(\beta\gamma)+40(\bar{\gamma}\bar{\alpha})(\gamma\alpha)
\end{aligned}
$$

where $\alpha^i,\beta^i,\gamma^i,\alpha,\beta,\gamma$ are the scalar components of $A^i,B^i,C^i,A,B,C$.

Superpotential: $W(A^i, B^j, C^k, A, B, C) = \sqrt{40} d_{ijk} A^i B^j C^k + \sqrt{40} ABC$

D-terms: $\frac{1}{2} D^\alpha D^\alpha + \frac{1}{2} D_1 D_1 + \frac{1}{2} D_2 D_2$ where:

$$D^\alpha = \frac{1}{\sqrt{3}} \left( \alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j \right)$$

$$D_1 = \frac{\sqrt{10}}{3} \left( \alpha^i (3\delta_i^j) \alpha_j + \bar{\alpha}(3)\alpha + \beta^i (-3\delta_i^j) \beta_j + \bar{\beta}(-3)\beta \right)$$

$$D_2 = \frac{\sqrt{40}}{3} \left( \alpha^i (\frac{1}{2}\delta_i^j) \alpha_j + \bar{\alpha}(\frac{1}{2})\alpha + \beta^i (\frac{1}{2}\delta_i^j) \beta_j + \bar{\beta}(\frac{1}{2})\beta + \gamma^i (-1\delta_i^j) \gamma_j + \bar{\gamma}(-1)\gamma \right)$$

Soft scalar supersymmetry breaking terms, $\mathcal{L}^{SSB}_{scalar}$:

$$\left( \frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \alpha^i \alpha_i + \left( \frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \bar{\alpha}\alpha + \left( \frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \beta^i \beta_i +$$
$$\left( \frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \bar{\beta}\beta + \left( \frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_2^3} \right) \gamma^i \gamma_i + \left( \frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \bar{\gamma}\gamma +$$
$$\sqrt{2}80 \left[ \left( \frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) d_{ijk} \alpha^i \beta^j \gamma^k + \left( \frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) \alpha\beta\gamma + h.c. \right],$$

Gaugino mass, $M = (1 + 3\tau) \frac{R_1^2 + R_2^2 + R_3^2}{8\sqrt{R_1^2 R_2^2 R_3^2}}$, $\tau$ torsion coeff.

Potential, $V = V_F + V_D + V_{soft}$

# The Wilson flux breaking

$M^4 \times B_o \to M^4 \times B$, $B = B_o/F^{S/R}$

$F^{S/R}$ - a freely acting discrete symmetry of $B_o$.

1. $B$ becomes multiply connected
2. For every element $g \in F^{S/R}$,

$$\rightsquigarrow \mathcal{V}_g = Pexp\left(-i\int_{\gamma_g} T^a A^a_M(x)dx^M\right) \in H$$

3. If the contour is non-contractible $\rightsquigarrow$ $\mathcal{V}_g \neq 1$ and then $f(g(x)) = \mathcal{V}_g f(x)$, which leads to a breaking of $H$ to $K' = C_H(T^H)$, where $T^H$ is the image of the homomorphism of $F^{S/R}$ into $H$.
4. Matter fields invariant under $F^{S/R} \oplus T^H$.

In the case of $SU(3)/U(1) \times U(1)$ a freely acting discrete group is:

$$F^{S/R} = \mathbb{Z}_3 \subset W\,, W = \frac{W_S}{W_R}\,,$$

$W_{S,R}$: Weyl group of $S$, $R$.

$$\rightsquigarrow \gamma_3 = \mathrm{diag}(\mathbb{1}, \omega\,\mathbb{1}, \omega^2\,\mathbb{1}), \quad \omega = e^{2i\pi/3} \in \mathbb{Z}_3$$

The fields that are invariant under $F^{S/R} \oplus T^H$ survive, i.e.:

$$\begin{array}{l} A_\mu = \gamma_3 A_\mu \gamma_3^{-1} \\ A^i = \gamma_3 A^i\,, \quad B^i = \omega\gamma_3 B^i\,, \quad C^i = \omega^2\gamma_3 C^i \\ A = A\,, \quad B = \omega B\,, \quad C = \omega^2 C \end{array}$$

$$\rightsquigarrow \mathcal{N} = 1\,, \quad SU(3)_c \times SU(3)_L \times SU(3)_R\,,$$

Recall that $\quad 27 = (1, 3, \overline{3}) + (\overline{3}, 1, 3) + (3, \overline{3}, 1)$

with matter superfields in:

$$\begin{array}{ccc} (1,3,\overline{3})_{(3,1/2)}, & (\overline{3},1,3)_{(-3,1/2)}, & (3,\overline{3},1)_{(0,-1)} \\ \updownarrow & \updownarrow & \updownarrow \\ L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & S \end{pmatrix}, & q^c = \begin{pmatrix} d_R^{c1} & u_R^{c1} & D_R^{c1} \\ d_R^{c2} & u_R^{c2} & D_R^{c2} \\ d_R^{c3} & u_R^{c3} & D_R^{c3} \end{pmatrix}, & Q = \begin{pmatrix} d_L^1 & d_L^2 & d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix} \end{array}$$

and the surviving singlet

$$\theta \rightarrow (1,1,1)_{(3,1/2)} \ .$$

Introducing appropriate non-trivial monopole charges in $U(1) \times U(1)$, 3 indentical flavours can appear for each of the chiral superfields.

# Further Gauge Breaking of $SU(3)^3$

*Babu - He - Pakvasa '86; Ma - Mondragon - Z '04;*
*Leontaris - Rizos '06; Sayre - Wiesenfeldt - Willenbrock '06*

At least two generations of $L$ acquire vevs that break the GUT:

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V & 0 & 0 \end{pmatrix}, \quad \langle L_s^{(2)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V \end{pmatrix}$$

each one alone is not enough to produce the (MS)SM gauge group:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)$$
$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)'_R \times U(1)'$$

Their combination gives the desired breaking:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y$$

Electroweak breaking then proceeds by:

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} \upsilon_d & 0 & 0 \\ 0 & \upsilon_u & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

# Effective Theory

*Polchinski '84*

If an effective $4D$ theory is renormalizable by power counting, it is consistent to consider it as a renormalizable theory.

→ We choose to respect the symmetries and model structure that are derived by the higher-dimensional theory and its dimensional reduction.
→ We treat all the parameters of the effective theory as free parameters, to the extend allowed by symmetries.

i.e.

- one coupling for all kinetic terms + $D$-terms
- one coupling for all superpotential terms
- independent couplings for soft SUSY + R-symmetry breaking terms (e.g. the $bL^3$ term included in the scalar potential which breaks the R-symmetry and will be necessary for the Split NMSSM phenomenology)

# Scales of Parameters

Choosing $M_{Comp} = M_{GUT}$:

– Soft trilinear terms $\sim \frac{1}{R_i} \sim \mathcal{O}(M_{GUT})$

– Soft scalar masses $\sim \frac{1}{R_i^2} \sim \mathcal{O}(M_{GUT}^2)$

– Specific choice of radii and soft parameters $\to m^2_{\theta_S^{(3)}} \sim \mathcal{O}(M_{EW}^2)$

We use vevs in all $L_s^{(i)}$ and $\theta^{(1,2)}$ to break the gauge group:

$$SU(3)^3 \times U(1)^2 \xrightarrow{V} SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{v_{u,d}} SU(3)_c \times U(1)_{em}$$

- $\mu$ terms for each generation of Higgs doublets emerge radiatively since R-symmetry is now broken: $H_u^{(i)} H_d^{(i)} \overline{\theta}^{(i)}$
- Lepton Yukawa terms that were forbidden by the R-symmetry emerge similarly: $L\overline{e}H_d$
- A $\mathcal{Z}_2$ discrete symmetry makes sure we have no dangerous terms
- Taking into account the rest of the allowed terms
  $\to$ the $S$, $\nu_R$ and $\theta^{(1,2)}$ fermions become supermassive
  $\to$ $\theta^{(3)}$ fermion acquires an $m \sim \mathcal{O}(EW)$ due to cancellation among vevs

# Low Energy Effective Model

| | |
|---|---|
| $D$, $S$, $\nu_R$ masses | $\mathcal{O}(GUT)$ |
| sfermion masses | $\mathcal{O}(GUT)$ |
| soft trilinear couplings / soft B term | $\mathcal{O}(GUT)$ |
| soft Higgs mass parameters | $\mathcal{O}(GUT)$ |
| fermion & scalar $\theta^{(1,2)}$ masses | $\mathcal{O}(GUT)$ |
| fermion & scalar $\theta^{(3)}$ masses | $\mathcal{O}(EW)$ |
| unified gaugino mass | $\mathcal{O}(EW)$ |

- Trilinear term $H_u^{(3)} H_d^{(3)} \theta^{(3)} \xrightarrow{<\theta^{(3)}>\sim\mathcal{O}(EW)}$ light $\mu$-term
- $\theta^{(3)}$ singlet superfield & gauginos $\rightarrow$ light
- soft Higgs mass parameters, effective soft B-parameter and sfermions $\rightarrow$ superheavy $\rightarrow$ light Higgs mass

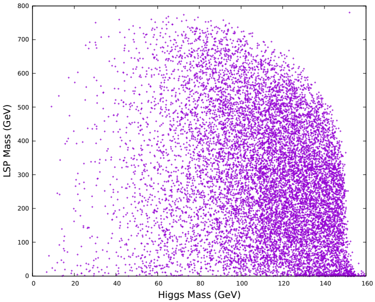
$\rightarrow$ **Split NMSSM**

*Demidov - Gorbunov '07;*
*Gabelmann - Mühlleitner - Staub '19*

# Phenomenological Analysis

2-loop Analysis (using SPheno):

- ✓ Particle content permits gauge unification
- ✓ Top, bottom and tau masses in agreement with the latest LHC measurements
- ✓ Light Higgs mass in agreement with the latest LHC measurements
- ✓ Light SUSY spectrum consistent with non-observation
- ✓ Stable LSP neutralino (CDM candidate)



# CSDR and the Einstein-Yang-Mills system

EYM theory with cosmological constant in $4 + d$ dimensions:

$$L = -\frac{1}{16\pi G}\sqrt{-g}R^{(D)} - \frac{1}{4g^2}\sqrt{-g}F^a_{MN}F^{aMN} - \sqrt{-g}\Lambda$$

The corresponding equations of motion are:

$$D_M F^{MN} = 0\,, \quad R_{MN} - \frac{1}{2}Rg_{MN} = -8\pi G T_{MN}$$

Spontaneous compactification: Solutions of the coupled EYM system corresponding to $M^4 \times B$ - $B$ a coset space and $\alpha, \beta$ coset indices + demanding $M^4$ to be flat Minkowski:

$$\Lambda = \frac{1}{4}\mathrm{Tr}\big(F_{\alpha\beta}F^{\alpha\beta}\big)$$

$\Lambda$ is absent in 4 dims: eliminates the vacuum energy of the gauge fields

$\Lambda$ equal to the minimum of the potential of the theory

# The potential of the reduced low-energy limit of 10-d heterotic string over $SU(3)/U(1) \times U(1)$

Low-energy effective action of $E_8 \times E_8$ heterotic string (bos part):

$$\mathcal{S}_{het} = \frac{1}{2\kappa^2} \int \mathrm{d}^{10}x \sqrt{-|g|} \left( R - \frac{1}{2} \partial_M \tilde{\Phi} \partial^M \tilde{\Phi} - \frac{e^{-\tilde{\Phi}}}{12} \tilde{H}_{MN\Lambda} \tilde{H}^{MN\Lambda} + \frac{\alpha' e^{-\frac{1}{2}\tilde{\Phi}}}{4} \mathrm{Tr}\, F_{MN} F^{MN} \right)$$

- $\kappa^2 = 8\pi G^{(10)}$ the 10-d gravitational constant
- $\alpha'$ the Regge slope parameter
- $R$ the Ricci scalar of the 10-d (target) space
- $\tilde{\Phi}$ the dilaton scalar field
- $\tilde{H}$ the field strength tensor of the 2-form $B_{MN}$ field
- $F$ the field strength tensor of the $E_8 \times E_8$ gauge field

Also, $g_s^2 = e^{2\tilde{\Phi}_0}$ is the string coupling constant ($\tilde{\Phi}_0$ is the constant mode of the dilaton)

Application of the CSDR over $SU(3)/U(1) \times U(1)$ leads to a $4-d$ scalar potential *Chatzistavrakidis - Z '09*

The contributions of the three sectors after the CSDR:

$$V_{gr} = -\frac{1}{4\kappa^2} e^{-\tilde{\phi}} \left( \frac{6}{R_1^2} + \frac{6}{R_2^2} + \frac{6}{R_3^2} - \frac{R_1^2}{R_2^2 R_3^2} - \frac{R_2^2}{R_1^2 R_3^2} - \frac{R_3^2}{R_1^2 R_2^2} \right)$$

$$V_H = \frac{1}{2\kappa^2} e^{-\tilde{\phi}} \left[ \frac{(b_1^2 + b_2^2 + b_3^2)^2}{(R_1 R_2 R_3)^2} + \sqrt{2} i \alpha' \frac{1}{R_1 R_2 R_3} (b_1^2 + b_2^2 + b_3^2)(d_{ijk} \alpha^i \beta^j \gamma^k - h.c.) \right]$$

$$V_F = \frac{\alpha'}{8\kappa^2} e^{-\frac{\tilde{\phi}}{2}} \left[ c + \left( \frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \alpha^i \alpha_i + \left( \frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \beta^i \beta_i + \left( \frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \gamma^i \gamma_i \right.$$
$$+ \sqrt{2} 80 \frac{R_1^2 + R_2^2 + R_3^2}{R_1 R_2 R_3} (d_{ijk} \alpha^i \beta^j \gamma^k + h.c.) + \frac{1}{6} \left( \alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j \right)^2$$
$$+ 5 \left( \alpha^i \alpha_i - \beta^i \beta_i \right)^2 + \frac{10}{3} \left( \alpha^i \alpha_i + \beta^i \beta_i - 2 \gamma^i \gamma_i \right)^2$$
$$\left. + 40 \alpha^i \beta^j d_{ijk} d^{klm} \alpha_l \beta_m + 40 \beta^i \gamma^j d_{ijk} d^{klm} \beta_l \gamma_m + 40 \alpha^i \gamma^j d_{ijk} d^{klm} \alpha_l \gamma_m \right]$$

*Possible compensation to the negative gravity contribution by the presence of gauge and 3-form sectors.*

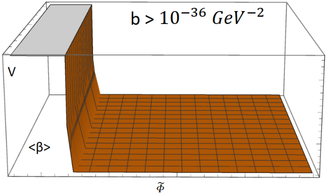
*Gibbons '84; De Wit - Smit - Dass '87;*
*Maldacena - Nuñez '01, Manousselis - Prezas - Z '06*

# Results

*Manolakos - Patellis - Z '22*

- Indicative results for the case

$$\begin{array}{ccccc} E_8 & \supset & G_2 & \times & F_4 \\ & & \cup & & \cap \\ E_8 & \supset & SU_3 & \times & E_6 \end{array}$$




where $\beta$ is the vev-acquiring scalar and $b$ is a parameter of the 3-form potential.

- Working on the case

$$\begin{array}{ccccccc} E_8 & \supset & SU_3 & \times & E_6 & & \\ & & \cup & & \cap & & \\ E_8 & \supset & U_1^2 & \times & E_6 & \times & U_1^2 \end{array}$$

we find similar behaviour for $\Sigma b_i > 10^{-33} GeV^{-2}$ , i.e. before the Wilson flux and other breakings.

✓There is a slice of parameter space for Minkowski vacua.

*THANK YOU!*



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